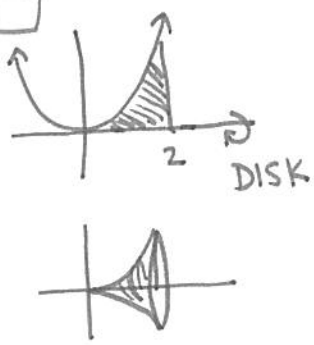
DATE: _____

VOLUME (Disk & Washer) MULTIPLE-CHOICE

1. Find the volume of the solid bounded by $y = x^2$, $x = 2$, and $y = 0$, revolved about the x-axis.

(A) $\frac{64\pi}{3}$ (B) 8π (C) $\frac{8\pi}{3}$ (D) $\frac{128\pi}{5}$ **(E) $\frac{32\pi}{5}$**

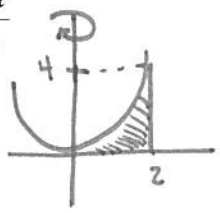
$$V = \pi \int_0^2 (x^2)^2 \, dx$$
$$= \pi \int_0^2 x^4 \, dx$$
$$= \pi \left(\frac{1}{5}x^5\right)\Big|_0^2$$
$$= \pi \left(\frac{32}{5} - 0\right) = \frac{32\pi}{5}$$



2. Find the volume of the solid bounded by $y = x^2$, $x = 2$, and $y = 0$, revolved about the y-axis.

(A) $\frac{16\pi}{3}$ (B) 4π (C) $\frac{32\pi}{5}$ **(D) 8π** (E) $\frac{8\pi}{3}$

$$V = \pi \int_0^4 (2^2 - (\sqrt{y})^2) \, dy$$
$$= \pi \int_0^4 (4 - y) \, dy$$
$$= \pi \left(4y - \frac{1}{2}y^2\right)\Big|_0^4$$
$$= \pi (16 - 8 - 0)$$
$$= 8\pi$$

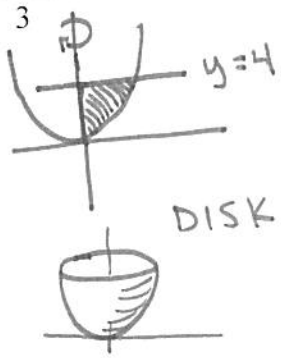


3. Find the volume of the solid bounded by $y = x^2$, $x = 0$, $y = 0$, and $y = 4$, revolved about the y-axis.

$\sqrt{y} = x$

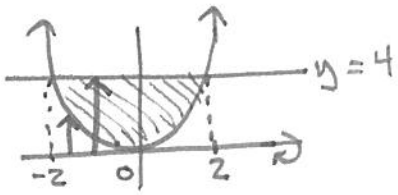
(A) 8π (B) 4π (C) $\frac{64\pi}{3}$ (D) $\frac{32\pi}{3}$ (E) $\frac{16\pi}{3}$

$$V = \pi \int_0^4 (\sqrt{y})^2 \, dy$$
$$= \pi \int_0^4 y \, dy$$
$$= \pi \left(\frac{1}{2}y^2\right)\Big|_0^4$$
$$= \pi (8 - 0)$$
$$= 8\pi$$

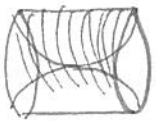


4. Find the volume of the solid bounded by $y = x^2$, $y = 4$, revolved about the x-axis.

(A) $\frac{64\pi}{5}$ (B) $\frac{512\pi}{15}$ **(C) $\frac{256\pi}{5}$** (D) $\frac{128\pi}{5}$ (E) none of these



WASHER



$$V = \pi \int_0^2 (4^2 - (x^2)^2)\, dx$$

$$= \pi \int_0^2 (16 - x^4)\, dx$$

$$= \pi \left(16x - \frac{1}{5}x^5\right)\Big|_0^2$$

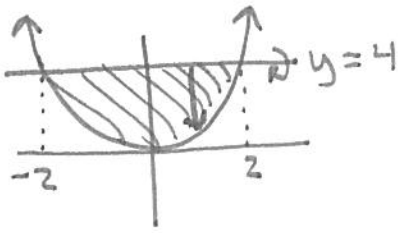
$$= \pi \left(32 - \frac{32}{5} - 0\right)$$

$$= \frac{128}{5}\pi$$

double the volume $2\left(\frac{128}{5}\pi\right) = \frac{256}{5}\pi$

5. Find the volume of the solid bounded by $y = x^2$, $y = 4$, revolved about the *line* $y = 4$.

(A) $\frac{256\pi}{15}$ (B) $\frac{256\pi}{5}$ (C) $\frac{512\pi}{5}$ **(D) $\frac{512\pi}{15}$** (E) $\frac{64\pi}{3}$



DISK

$$V = \pi \int_{-2}^{2} (4 - x^2)^2\, dx$$

$$= 2\pi \int_0^2 (16 - 8x^2 + x^4)\, dx$$

$$= 2\pi \left(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right)\Big|_0^2$$

$$= 2\pi \left(32 - \frac{64}{3} + \frac{32}{5}\right) - 0$$

$$= 2\pi \left(32 - \frac{320}{15} + \frac{96}{15}\right)$$

$$= 2\pi \left(32 - \frac{224}{15}\right)$$

$$= 2\pi \left(\frac{256}{15}\right) = \frac{512}{15}\pi$$