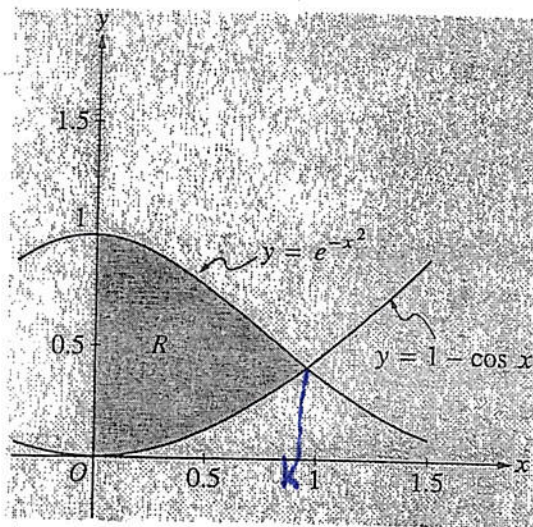A graphing calculator is required for some problems or parts of problems.


$k = 0.942$

1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

   (a) Find the area of the region R.

   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

a) Area of $R = \int_0^k \left(e^{-x^2} - (1 - \cos x) \right) dx$

$= 0.591$

b) Volume $= \pi \int_0^k \left((e^{-x^2})^2 - (1 - \cos x)^2 \right) dx$

$= 1.747$

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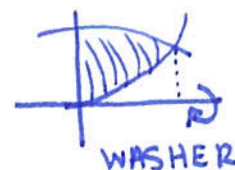
c) $V = \int_0^k (\text{side})^2 dx$

$= \int_0^k \left(e^{-x^2} - (1 - \cos x) \right)^2 dx$

$= 0.461$

$\text{Area of } \square = (\text{side})^2$

$= \left(e^{-x^2} - (1 - \cos x) \right)^2$
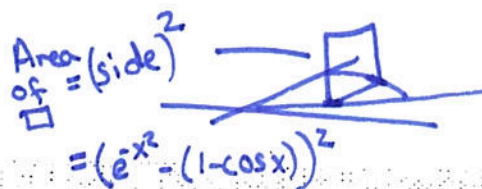
Question 1

Let R be the region bounded by the y-axis and the graphs of $y = \dfrac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.
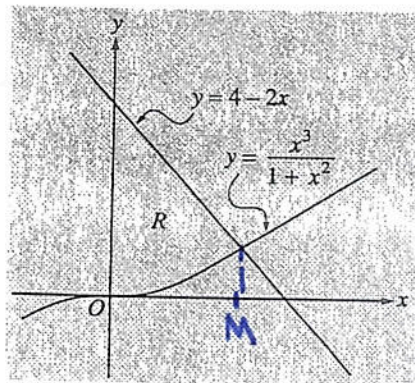
(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

$M = 1.488$

a) Area of $R = \displaystyle\int_0^M \left(4-2x - \frac{x^3}{1+x^2}\right) dx$

$\quad = 3.215$

b) Volume $= \pi \displaystyle\int_0^M \left((4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right) dx$

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$\quad = 31.885$

c) $V = \displaystyle\int_0^M (\text{side})^2\, dx$

$\quad = \displaystyle\int_0^M \left(4-2x - \frac{x^3}{1+x^2}\right)^2 dx$

Area $= (\text{side})^2$

$= \left(4-2x - \frac{x^3}{1+x^2}\right)^2$

$\quad = 8.997$