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Arc Length Practice

1. The length of the curve $y = \ln(\sec x)$ from $x = 0$ to $x = b$, where $0 < b < \frac{\pi}{2}$, may be expressed by which of the following integrals?

$\boxed{(A)}$ $\int_0^b \sec x\, dx$

(B) $\int_0^b \sec^2 x\, dx$

(C) $\int_0^b (\sec x \tan x)\, dx$

(D) $\int_0^b \sqrt{1+(\ln(\sec(x))^2}\, dx$

(E) $\int_0^b \sqrt{1+(\sec^2 x \tan^2 x)}\, dx$

$$\frac{dy}{dx} = \frac{1}{\sec x} \cdot \sec x \tan x$$
$$= \tan x$$
$$L = \int_0^b \sqrt{1+(\tan x)^2}\, dx \qquad \tan^2 x + 1 = \sec^2 x$$
$$= \int_0^b \sqrt{\sec^2 x}\, dx$$
$$= \int_0^b \sec x\, dx$$

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2. The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by:

(A) $\int_0^2 \sqrt{1 + x^6}\, dx$

(B) $\int_0^2 \sqrt{1 + 3x^2}\, dx$

(C) $\pi \int_0^2 \sqrt{1 + 9x^4}\, dx$

(D) $2\pi \int_0^2 \sqrt{1 + 9x^4}\, dx$

$\boxed{(E)}$ $\int_0^2 \sqrt{1 + 9x^4}\, dx$

$$\frac{dy}{dx} = 3x^2$$
$$L = \int_0^2 \sqrt{1+(3x^2)^2}\, dx$$
$$= \int_0^2 \sqrt{1 + 9x^4}\, dx$$

3. What is the length of the arc of $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 3$?

(A) $\frac{8}{3}$

(B) 4

$\boxed{(C)}$ $\frac{14}{3}$

(D) $\frac{16}{3}$

(E) 7

$$\frac{dy}{dx} = x^{1/2}$$
$$L = \int_0^3 \sqrt{1+(x^{1/2})^2}\, dx$$
$$= \int_0^3 \sqrt{1+x}\, dx \qquad u = 1+x \quad du = dx \qquad u(0) = 1 \quad u(3) = 4$$
$$= \int_0^3 (1+x)^{1/2}\, dx$$
$$= \int_1^4 u^{1/2}\, du = \frac{2}{3}u^{3/2}\Big|_1^4 = \frac{2}{3}(4^{3/2} - 1) = \frac{2}{3}(8) - \frac{2}{3} = \boxed{\frac{14}{3}}$$

4. The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1 + 9x^4}\, dx$. If the curve contains the point (1,6), which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$

$\boxed{(B)}$ $y = 5 + x^3$

(C) $y = 6 + x^3$

(D) $y = 6 - x^3$

(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$

$$L = \int_1^4 \sqrt{1 + (\underbrace{3x^2}_{})^2}\, dx$$
$$\frac{dy}{dx} = 3x^2$$
$$\int dy = \int 3x^2\, dx$$
$$y = x^3 + C$$
$$6 = 1^3 + c \qquad \boxed{y = x^3 + 5}$$

5. Find the exact length of the given curve $y = x^{3/2}$ from $x = 0$ to $x = 3$.

$$L = \int_0^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad \frac{dy}{dx} = \frac{3}{2}x^{1/2}$$

$$= \int_0^3 \sqrt{1 + \left(\tfrac{3}{2}x^{1/2}\right)^2}\, dx$$

$$= \int_0^3 \sqrt{1 + \tfrac{9}{4}x}\, dx \qquad u = 1 + \tfrac{9}{4}x \qquad u(0) = 1$$

$$\frac{du}{dx} = \frac{9}{4} \qquad u(3) = 1 + \tfrac{27}{4} = \tfrac{31}{4}$$

$$= \frac{4}{9}\int_1^{31/4} u^{1/2}\, du \qquad \tfrac{4}{9}\, du = dx$$

$$= \frac{4}{9}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^{31/4}$$

$$= \boxed{\frac{8}{27}\left(31^{3/2} - 1\right)}$$

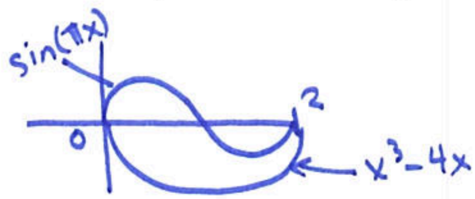
6. Find the exact length of the given curve $y = \frac{3}{4}x^{4/3} - \frac{3}{8}x^{2/3}$ from $x = 1$ to $x = 8$.

$$\frac{dy}{dx} = x^{1/3} - \tfrac{1}{4}x^{-1/3}$$

$$L = \int_1^8 \sqrt{1 + \left(x^{1/3} - \tfrac{1}{4}x^{-1/3}\right)^2}\, dx$$

$$= 12.375$$

7. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$ on $[0,2]$. Find the perimeter of the region R.



$$\frac{dy}{dx} = \pi\cos(\pi x) \qquad \frac{dy}{dx} = 3x^2 - 4$$

$$L = \int_0^2 \sqrt{1 + (\pi\cos\pi x)^2}\, dx \qquad L = \int_0^2 \sqrt{1 + (3x^2 - 4)^2}\, dx$$

$$= 4.610 \qquad = 6.634$$

perimeter $= 4.610 + 6.634$

$= 11.244$