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7.4 Partial Fractions (Target 8G)

Let's do two more examples involving partial fractions.

Multiply every term by LCD

Decompose each fraction.

1. $\frac{-x^2+2x-5}{(x+1)(x^2+6x+9)} = \frac{-x^2+2x-5}{(x+1)(x+3)(x+3)} = \frac{-x^2+2x-5}{(x+1)(x+3)^2} = \frac{A}{x+1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

LCD: $(x+1)(x+3)^2$. Multiply both sides by LCD to get...

$$-x^2+2x-5 = A(x+3)^2 + B(x+1)(x+3) + C(x+1)$$
$$-x^2+2x-5 = A(x+3)(x+3) + B(x+1)(x+3) + C(x+1)$$
$$-x^2+2x-5 = A(x^2+6x+9) + B(x^2+4x+3) + Cx+C$$
$$-x^2+2x-5 = Ax^2+6Ax+9A+Bx^2+4Bx+3B+Cx+C$$
$$-x^2+2x-5 = Ax^2+Bx^2+6Ax+4Bx+Cx+9A+3B+C$$
$$-1x^2+2x-5 = (A+B)x^2 + (6A+4B+C)x + 9A+3B+C$$

$\therefore$ $-1 = A+B$, $\left\{\begin{array}{l} 2 = 6A+4B+C \\ -5 = 9A+3B+C \end{array}\right.$ Can set up matrix $\left[\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 6 & 4 & 1 & 2 \\ 9 & 3 & 1 & -5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 10 \end{array}\right]$.

\- OR -

$$-1(2) = -1(6A+4B+C)$$
$$-5 = 9A+3B+C$$

$\downarrow$

$$-2 = -6A-4B-\cancel{C}$$
$$-5 = 9A+3B+\cancel{C}$$
$$-7 = 3A-B$$

$$-1 = A+\cancel{B}$$
$$-7 = 3A-\cancel{B}$$
$$-8 = 4A$$
$$\boxed{-2 = A}$$

$$-1 = A+B$$
$$-1 = -2+B$$
$$+2 \quad +2$$
$$\boxed{1 = B}$$

$$2 = 6A+4B+C$$
$$2 = 6(-2)+4(1)+C$$
$$2 = -12+4+C$$
$$2 = -8+C$$
$$\boxed{10 = C}$$

$$\therefore \frac{-x^2+2x-5}{(x+1)(x^2+6x+9)} = \frac{-2}{x+1} + \frac{1}{x+3} + \frac{10}{(x+3)^2} \checkmark$$

2. $\frac{x}{x^2-1} = \frac{x}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$. Multiply by LCD: $(x+1)(x-1)$. We set:

$$x = A(x-1) + B(x+1)$$

Let $x = 1$. Then

$$1 = A(1-1) + B(1+1)$$
$$1 = 2B$$
$$\boxed{\tfrac{1}{2} = B}$$

Let $x = -1$. Then

$$-1 = A(-1-1) + B(-1+1)$$
$$-1 = -2A$$
$$\boxed{\tfrac{1}{2} = A}$$

$$\therefore \frac{x}{x^2-1} = \frac{\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$$
$$= \frac{1}{2(x+1)} + \frac{1}{2(x-1)}$$
$$= \frac{1}{2}\left(\frac{1}{x+1} + \frac{1}{x-1}\right) \checkmark$$

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7.4 Mountain Climber

See final solution on next page

5) Find the partial fraction decomposition: $\frac{3x+5}{x^3+4x^2+5x+2}$

$x = \frac{\pm p}{q} = \frac{\pm 1, \pm 2}{\pm 1} = \pm 1, \pm 2$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 5 & 2 \\ & & 1 & 5 & 5 \\ \hline & 1 & 5 & 10 & 1 \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & 4 & 5 & 2 \\ & & -1 & -3 & -2 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

x^2+3x+2

$(x+1)(x+2)$

$$\frac{3x+5}{(x+1)(x+1)(x+2)} = \frac{3x+5}{(x+2)(x+1)^2} = \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$3x+5 = A(x+1)(x+1) + B(x+1)(x+2) + C(x+2)$

$3x+5 = Ax^2 + 2Ax + A + Bx^2 + 3Bx + 2B + Cx + 2C$

$3x+5 = (A+B)x^2 + (2A+3B+C)x + (A+2B+2C)$

$0 = A+B \Rightarrow A = -B$ $\quad \boxed{A=-1}$

$3 = 2A+3B+C \Rightarrow 3 = -2B+3B+C$

$\boxed{3 = B+C} \rightarrow 3 = B+2$, $\boxed{1=B}$

$5 = A+2B+2C$

$5 = -B+2B+2C$

$\boxed{5 = B+2C}$

$3 = B+C$

$-(5 = B+2C)$

$-2 = -C \Rightarrow \boxed{2 = C}$

4) Find the partial fraction decomposition: $\frac{3x^3+6x-1}{(x^2+2)^2}$

$$\frac{3x^3+6x-1}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2}$$

$3x^3+6x-1 = (Ax+B)(x^2+2) + Cx+D$

$= Ax^3 + 2Ax + Bx^2 + 2B + Cx + D$

$= Ax^3 + Bx^2 + 2Ax + Cx + 2B + D$

$= Ax^3 + Bx^2 + (2A+C)x + (2B+D)$

$\therefore \boxed{3 = A}$

$\boxed{0 = B}$

$6 = 2A + C$

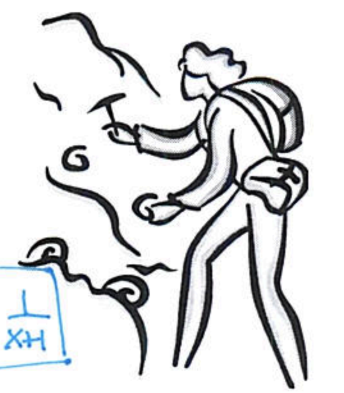
$-1 = 2B + D \Rightarrow \boxed{-1 = D}$

$6 = 2(3) + C$

$\boxed{0 = C}$

$$\therefore \frac{3x^3+6x-1}{(x^2+2)^2} = \boxed{\frac{3x}{x^2+2} - \frac{1}{(x^2+2)^2}}$$

3) Find the partial fraction decomposition: $\frac{3x-2}{x^2-3x-4} = \frac{3x-2}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$



$3x-2 = A(x+1) + B(x-4)$

Let $x = -1$. Then

$-3-2 = A(-1+1) + B(-1-4)$

$-5 = -5B$

$\boxed{1 = B}$

Let $x = 4$. Then

$12-2 = A(4+1) + B(4-4)$

$10 = 5A$

$\boxed{2 = A}$

$$\boxed{\therefore \frac{3x-2}{x^2-3x-4} = \frac{2}{x-4} + \frac{1}{x+1}}$$

2) Find the partial fraction decomposition: $\frac{-6}{x^2-3x}$

$$\frac{-6}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$$

$-6 = A(x-3) + Bx$

Let $x = 3$. Then

$-6 = A(3-3) + B(3)$

$-6 = 3B$

$\boxed{-2 = B}$

Let $x = 0$. Then

$-6 = A(0-3) + B(0)$

$-6 = -3A$

$\boxed{2 = A}$

$$\boxed{\therefore \frac{-6}{x(x-3)} = \frac{2}{x} - \frac{2}{x-3}}$$

1) Write the terms for the partial fraction decomposition of the rational function:

$$\frac{x^4-4x^3+x-3}{x^2(x+4)^2(x^2+3)} = \boxed{\frac{Ax+B}{x^2} + \frac{C}{x+4} + \frac{D}{(x+4)^2} + \frac{Ex+F}{x^2+3}}$$

See solutions on next page or

Solutions:

1. $= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+4)} + \frac{D}{(x+4)^2} + \frac{Ex+F}{x^2+3}$

2. $= \frac{-2}{x-3} + \frac{2}{x}$

3. $= \frac{1}{x+1} + \frac{2}{x-4}$

4. $= \frac{3x}{x^2+2} + \frac{-1}{(x^2+2)^2}$

5. $= \frac{-1}{x+2} + \frac{1}{x+1} + \frac{2}{(x+1)^2}$