

8.1 Circles & Parabolas

Target 4A: Investigate the geometric properties of parabolas (vertex, focus, and directrix)

Target 4B: Derive the standard equation of a parabola and graph given two or three criteria

Review of Prior Concepts

<https://youtu.be/yaSpxDdKEAs>

What do you know about Conic Sections?



ANSWERS VARY
(circles, parabolas, ellipses, hyperbolas)

More Practice

Introduction to Conics

<https://www.khanacademy.org/math/algebra2/intro-to-conics-alg2><http://www.coolmath.com/algebra/25-conic-sections/01-introduction-circles-01><https://www.mathsisfun.com/geometry/conic-sections.html><https://www.youtube.com/watch?v=GDHNoQHQtQ><https://www.youtube.com/watch?v=SKNybVUuPXA>

SAT Connection

Passport to Advanced Math

12. Understand a nonlinear relationship between two variables

Example: Which of the following is an equation of a circle in

the xy -plane with center $(0, 4)$ and a radius withendpoint $(\frac{4}{3}, 5)$?

(A) $x^2 + (y - 4)^2 = \frac{25}{9}$

B) $x^2 + (y + 4)^2 = \frac{25}{9}$

C) $x^2 + (y - 4)^2 = \frac{5}{3}$

D) $x^2 + (y + 4)^2 = \frac{3}{5}$

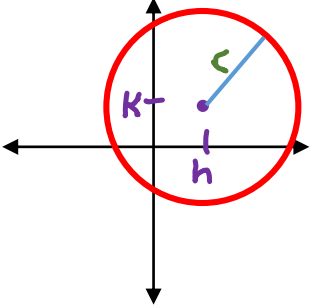
$$\begin{aligned} & (x-h)^2 + (y-k)^2 = r^2 \\ & (\frac{4}{3} - 0)^2 + (5 - 4)^2 = r^2 \\ & (\frac{4}{3})^2 + 1^2 = r^2 \\ & \frac{25}{9} = r^2 \\ & (x-0)^2 + (y-4)^2 = \frac{25}{9} \\ & x^2 + (y-4)^2 = \frac{25}{9} \end{aligned}$$

Solution

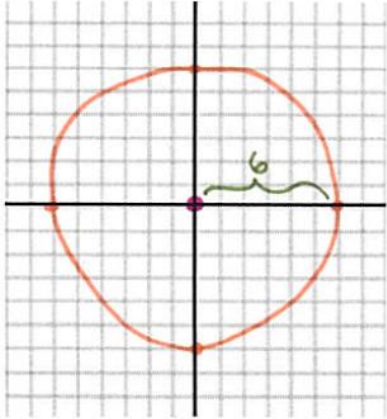
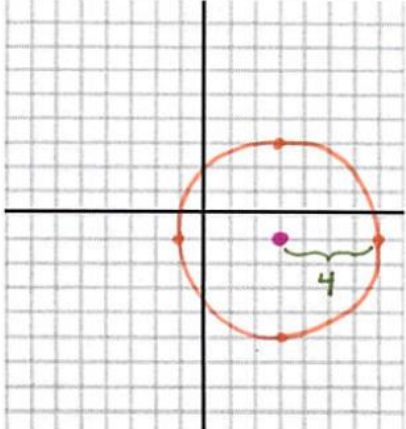
Circle

Definition (in your own words)

Set of all points equidistant from a given point
(equal distance)

Standard Form of Equation	$(x - h)^2 + (y - k)^2 = r^2$	<i>Sketch</i> 
Center	(h, k)	
Radius	r	
Eccentricity	$e = 0$	

Examples

Standard Form of Equation	$x^2 + y^2 = 36$	$(x - 3)^2 + (y + 1)^2 = 16$
Center	$(0, 0)$	$(3, -1)$
Radius	$r^2 = 36 \rightarrow r = 6$	$r^2 = 16 \rightarrow r = 4$
Eccentricity	$e = 0$	$e = 0$
Sketch		

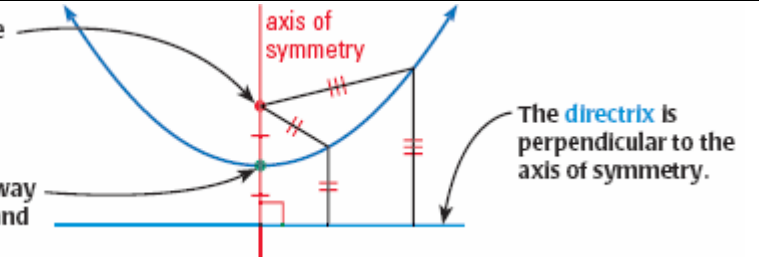
Parabola

Definition (in your own words)

Set of all points equidistant from a given point (focus) and a given line (directrix)
(point not on the line... ☹)

The **focus** lies on the axis of symmetry.

The **vertex** lies halfway between the focus and the directrix.



Vertical Axis (of Symmetry)		Sample Sketch	Example
Standard Form of Equation	$(x - h)^2 = 4p(y - k)$	$4p > 0$ 	$x^2 = -8(y - 2)$ vertex: $(0, 2)$ focus: $(0, 0)$ directrix: $y = 4$ axis of symmetry: $x = 0$
Opening	up when $4p > 0$ down when $4p < 0$		$4p < 0$
Vertex	(h, k)		
Focus	$(h, k+p)$		
Directrix	$y = k - p$		
Axis of Symmetry	$x = h$		
Focal Length	length from focus to vertex or directrix to vertex p		
Focal Width	width of parabola thru the focus $ 4p $		
Eccentricity	$e = 1$		

Horizontal Axis (of Symmetry)		Sketch	Example
Standard Form of Equation	$(y - k)^2 = 4p(x - h)$	$4p > 0$ 	$y^2 = 12(x - 2)$ Vertex $(2, 0)$ focus $(5, 0)$ directrix $x = -1$ Axis of symmetry $y = 0$ $4p = 12$ $p = 3$
Opening	right when $4p > 0$ left when $4p < 0$		$4p < 0$
Vertex	(h, k)		
Focus	$(h+p, k)$		
Directrix	$x = h - p$		
Axis of Symmetry	$y = k$		
Focal Length	length from focus to vertex or directrix to vertex p		
Focal Width	width of parabola thru the focus $ 4p $		
Eccentricity	$e = 1$		

Unit 4 (Chapter 8): Conic Sections

Examples:

1. Find the vertex, focus, directrix, and focal width of the parabola: $(y + 4)^2 = 6x - 12$

vertex: $(2, -4)$

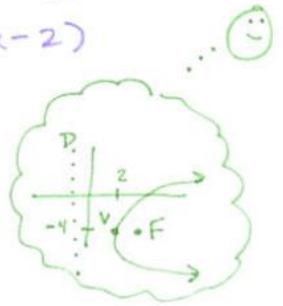
focus: $(2 + \frac{3}{2}, -4)$
 $(\frac{7}{2}, -4)$

directrix: $x = 2 - \frac{3}{2}$
 $x = \frac{1}{2}$

focal width: 6

$(y+4)^2 = 6(x-2)$

$6 = 4p$
 $\frac{6}{4} = p$
 $\frac{3}{2} = p$



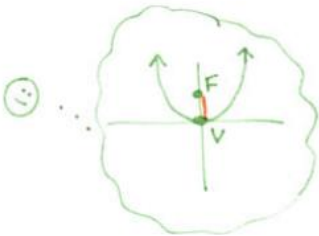
2. Find an equation in standard form for the parabola that satisfies the given conditions:

vertex $(0,0)$, focus $(0,2)$

$(x-h)^2 = 4p(y-k)$
 $(x-0)^2 = 4(2)(y-0)$

$x^2 = 8y$

$p \rightarrow$ distance b/n focus + vertex
 $p = 2$



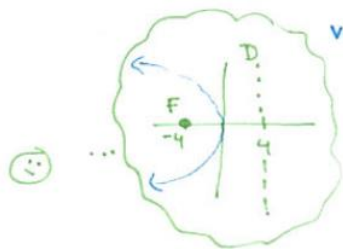
3. Find an equation in standard form for the parabola that satisfies the given conditions:

focus $(-4,0)$, directrix $x = 4$

vertex is midpt b/n
focus + directrix
Vertex: $(0,0)$

$p \rightarrow$ distance b/n focus + vertex
 $p = 4$
 \uparrow negative since parabola
opens left

$(y-k)^2 = 4p(x-h)$
 $(y-0)^2 = 4(-4)(x-0)$
 $y^2 = -16x$



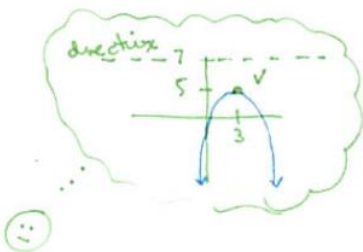
4. Find an equation in standard form for the parabola that satisfies the given conditions:

vertex $(3,5)$, directrix $y = 7$

$p \rightarrow$ distance b/n vertex +
directrix

$p = 2$
 \uparrow negative b/c parabola
opens down

$(x-h)^2 = 4p(y-k)$
 $(x-3)^2 = 4(-2)(y-5)$
 $(x-3)^2 = -8(y-5)$



5. What is the relationship between the vertex, focus, and directrix in the parabola?

The vertex is an equal distance from the focus as
it is from the directrix.

The distance from vertex to focus is p
and from vertex to directrix is p .

More Practice**Circles**

<http://www.regentsprep.org/regents/math/algtrig/atc1/circlelesson.htm>

<http://www.mathsisfun.com/algebra/circle-equations.html>

<http://www.mathwarehouse.com/geometry/circle/equation-of-a-circle.php>

<https://www.youtube.com/watch?v=FLM3x1qw3WY>

<https://www.youtube.com/watch?v=6r1GQCxyMKI>

Parabolas

<http://www.purplemath.com/modules/parabola3.htm>

<http://www.protutorcompany.com/finding-the-vertex-focus-and-directrix-of-a-parabola-given-in-standard-form/>

<https://braingenie.ck12.org/skills/108316>

<https://www.youtube.com/watch?v=qxjs3NHI5T8>

<https://www.youtube.com/watch?v=a3qUuSqxzBk>

Homework Assignment

p.639 #1,3,7-10all,11,13,15

SAT Connection**Solution**

Choice A is correct. The equation of a circle can be written as $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) are the coordinates of the center of the circle and r is the radius of the circle. Since the coordinates of the center of the circle are $(0, 4)$, the equation is $x^2 + (y - 4)^2 = r^2$, where r is the radius. The radius of the circle is the distance from the center, $(0, 4)$, to the given endpoint of a radius, $(\frac{4}{3}, 5)$. By the distance formula, $r^2 = \left(\frac{4}{3} - 0\right)^2 + (5 - 4)^2 = \frac{25}{9}$. Therefore, an equation of the given circle is $x^2 + (y - 4)^2 = \frac{25}{9}$.

Choice B is incorrect; it results from the incorrect equation $(x + h)^2 + (y + k)^2 = r^2$. Choice C is incorrect; it results from using r instead of r^2 in the equation for the circle. Choice D is incorrect; it results from using the incorrect equation $(x + h)^2 + (y + k)^2 = \frac{1}{r}$.