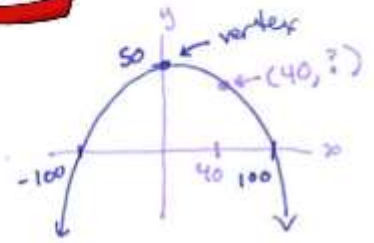
8.1, 8.2 & 8.3 Parabolas, Ellipses & Hyperbolas

Target 4A/4C/4E: Investigate the geometric properties of parabolas/ellipses/hyperbolas

Target 4B/4D/4F: Derive the standard equation of a parabola/ellipse/hyperbola and graph given two or three criteria

Practical Applications of Conic Sections

1. The parabolic arch shown in the figure is 50 feet above water at the center ar base. Will a boat that is 30 feet tall clear the arch 40 feet from the center?



vertex: (0, 50) point: (40, ?)
point: (100, 0) • (-100, 0)

$(x-h)^2 = 4p(y-k)$

$(x-0)^2 = 4p(y-50)$ ← sub in vertex

$x^2 = 4p(y-50)$

← sub in point (100, 0)

$100^2 = 4p(0-50)$ solve for p

$10000 = -200p$

$-50 = p$

$x^2 = 4(-50)(y-50)$

$x^2 = -200(y-50)$ ← use equation to find "?"

$40^2 = -200(y-50)$

$1600 = -200y + 10000$

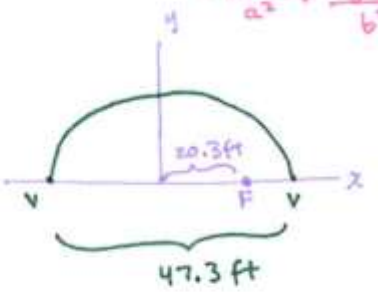
$-8400 = -200y$

$42 = y$ → height of bridge 40ft from center is 42ft.

∴, the 30 ft boat will clear the arch 40ft from the center

2. The Whispering Gallery in the Museum of Science and Industry in Chicago is 47.3 feet long. The distance from the center of the room to the foci is 20.3 feet. Find an equation that describes the shape of the room. How high is the room at its center?

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



major axis = 47.3ft

center to ~~from~~ foci: 20.3ft

b = ?

major axis ⇒ $2a = 47.3$

$a = 23.65$

center to focus ⇒ $c = 20.3$

$a^2 = b^2 + c^2$

$(23.65)^2 = b^2 + (20.3)^2$

$12.134 = b$

The room is 12.134 ft tall

$\frac{(x-0)^2}{23.65^2} + \frac{(y-0)^2}{12.134^2} = 1$

$\frac{x^2}{559.3225} + \frac{y^2}{147.2325} = 1$

3. A satellite is in elliptical orbit around the earth with the center of the Earth at one focus. The distance of the satellite from the Earth varies between 140 mi and 440 mi. Assume the Earth is a sphere with radius 3960 miles. Find an equation for the path of the satellite.

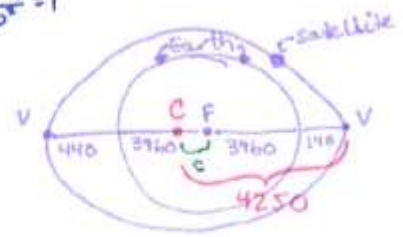
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

major axis = 440 + 3960 + 3960 + 140

= 8500 miles

$2a = 8500$

$a = 4250$ → distance from center to vertex



$c + 3960 + 140 = 4250$

$c + 4100 = 4250$

$c = 150$ → distance from focus to center

$a^2 = b^2 + c^2$

$4250^2 = b^2 + 150^2$

$18040000 = b^2$

$\frac{(x-0)^2}{4250^2} + \frac{(y-0)^2}{18040000} = 1$

$$\frac{x^2}{18062500} + \frac{y^2}{12252500} = 1$$

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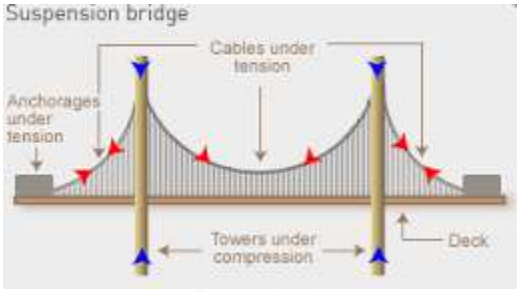
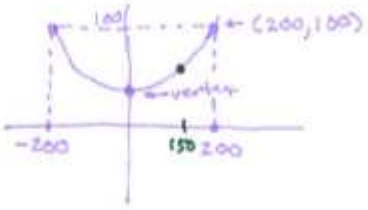
4. The towers of a suspension bridge are 400 feet apart and 100 feet high. Cables are at a height of 10 feet between the towers. Assume the *x*-axis is the road and the *y*-axis if the center of the bridge, write an equation for the parabola. What is the height of the cable at a point 50 feet from one of the towers?



vertex : (0, 10)

point : (200, 100)

point : (150, ?)



$(x-h)^2 = 4p(y-k)$

$(x-0)^2 = 4p(y-10)$ ← sub in vertex

$x^2 = 4p(y-10)$

$200^2 = 4p(100-10)$ ← sub in pt (200, 100)

$40000 = 360p$

$\frac{1000}{9} = p$

$x^2 = 4(\frac{1000}{9})(y-10)$

$150^2 = \frac{4000}{9}(y-10)$ ← use equation to find "?"

$\frac{9}{4000}(22500) = (\frac{4000}{9}(y-10))\frac{9}{4000}$

$50.625 = y - 10$

$60.625 = y$

The height of the cable is 60.625 ft

More Practice

Applications of Conic Sections

http://www3.ul.ie/~rynnet/swconics/applications_of_conic_sections.htm
https://www.youtube.com/watch?v=6GVumC_Pie0

Solving Real-World Conic Section Problems

http://www.shelovesmath.com/precal/parametrics-and-conics/#ApplicationsofEllipses
http://www.purplemath.com/modules/ellipse4.htm
https://www.youtube.com/watch?v=lg4uuxI8leE
https://www.youtube.com/watch?v=umqE1LeluOo
https://www.youtube.com/watch?v=T2eHdLyD4P4
https://www.youtube.com/watch?v=DB7IPSWbmhM

Homework Assignment

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