8.1, 8.2 & 8.3 Parabolas, Ellipses & Hyperbolas

Target 4A/4C/4E: Investigate the geometric properties of parabolas/ellipses/hyperbolas

Conics in General Form vs. Standard Form

How to write an ellipse equation from general form to standard form (or how to complete a square)

Steps	Example
Start with general form of equation.	$4x^2 + 9y^2 - 48x + 72y + 144 = 0$
Move the constant & group *x*-terms together and *y*-terms together.	$4x^2 - 48x + 9y^2 + 72y = -144$
Factor out the coefficient(s) on the square terms.	$4(x^2 - 12x) + 9(y^2 + 8y) = -144$
Leave empty space after the *x*-terms and the *y*-terms.	$4(x^2 \underbrace{- 12}x \qquad) + 9(y^2 \underbrace{+ 8}y \qquad) = -144$
Take ½ the coefficient of the linear terms and square that #.	$\frac{1}{2}(-12)$ $\qquad$ $\frac{1}{2}(8)$ $\left(\frac{1}{2}(-12)\right)^2 = 36$ $\qquad$ $\left(\frac{1}{2}(8)\right)^2 = 16$
Place the values into the empty spaces. Multiply the values by the coefficients and place on the other side of the equation.	$4(x^2 - 12x + 36) + 9(y^2 + 8y + 16) = -144 + \underbrace{4(36)} + \underbrace{9(16)}$
Write the *x*-terms and the *y*-terms in squared form (where the constant is ½ the coefficient of the linear terms).	$4(x - 6)^2 + 9(y + 4)^2 = 144$
Divide by value on right side to get equation into standard form.	$\frac{(x-6)^2}{36} + \frac{(y+4)^2}{16} = 1$

SAT Connection

Passport to Advanced Math

12. Understand a nonlinear relationship between two variables

Example:

$$x^2 + y^2 + 4x - 2y = -1$$

The equation of a circle in the *xy*-plane is shown above. What is the radius of the circle?

A) 2

B) 3

C) 4

D) 9

Solution

$x^2+4x+y^2-2y=-1$

$x^2+4x+4+y^2-2y+1=-1+4+1$

$(x+2)^2+(y-1)^2=4$

$r^2=4$

$r=2$

Now, you try….

Example 1:

Write the equation in standard form and identify the center, vertices, and foci.

$16x^2 + 4y^2 - 32x + 24y - 12 = 0$

$16x^2 - 32x + 4y^2 + 24y = 12$

$16(x^2 - 2x \quad) + 4(y^2 + 6y \quad) = 12$

$\left(\frac{1}{2}(-2)\right)^2$
$(-1)^2$
1

$\left(\frac{1}{2}(6)\right)^2$
$(3)^2$
9

$16(x^2 - 2x + 1) + 4(y^2 + 6y + 9) = 12 + 16(1) + 4(9)$

$12 + 16 + 36$
$28 + 36$
64

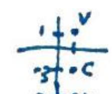
$16(x-1)^2 + 4(y+3)^2 = 64$

$\frac{16(x-1)^2}{64} + \frac{4(y+3)^2}{64} = \frac{64}{64}$

$$\boxed{\frac{(x-1)^2}{4} + \frac{(y+3)^2}{16} = 1}$$

center: $(1, -3)$
vertices: $(1, 1)$ and $(1, -7)$
foci: $(1, -3+\sqrt{12})$ and $(1, -3-\sqrt{12})$

$a^2 = 16$
$a = 4$



$a^2 = b^2 + c^2$
$16 = 4 + c^2$
$12 = c^2$
$\sqrt{12} = c$
or $2\sqrt{3}$

Write the equation in standard form and identify the center, vertices, and asymptotes.

$4x^2 - 5y^2 + 40x - 30y - 45 = 0$

$4x^2 + 40x - 5y^2 - 30y = 45$

$4(x^2 + 10x \quad) - 5(y^2 - 6y \quad) = 45$

$\left(\frac{1}{2}(10)\right)^2$
$(5)^2$
25

$\left(\frac{1}{2}(-6)\right)^2$
$(-3)^2$
9

don't forget the "-"

$4(x^2 + 10x + 25) - 5(y^2 - 6y + 9) = 45 + 4(25) - 5(9)$

$45 + 100 - 45$
100

$4(x+5)^2 - 5(y-3)^2 = 100$

$\frac{4(x+5)^2}{100} - \frac{5(y-3)^2}{100} = \frac{100}{100}$

$$\boxed{\frac{(x+5)^2}{25} - \frac{(y-3)^2}{20} = 1}$$

center: $(-5, 3)$
vertices: $(0, 3)$ and $(-10, 3)$
asymptotes: $y = \pm \frac{2\sqrt{5}}{5}(x+5) + 3$

$a^2 = 25$
$a = 5$

$b^2 = 20$
$b = \sqrt{20}$
or $2\sqrt{5}$

How to write a parabola equation from general form to standard form (or how to complete a square)

Steps	Example
answers vary by students	$x^2 - 6x - 12y - 3 = 0$ $x^2 - 6x = 12y + 3$ $x^2 - 6x + 9 = 12y + 3 + 9$ $(x-3)^2 = 12y + 12$ $(x-3)^2 = 12(y+1)$

Trade this paper with a classmate.
Your classmate will try to follow your steps for the example below.

Example

Write the equation in standard form and identify the vertex, focus, and directrix.

$y^2 + 4y + 8x + 12 = 0$

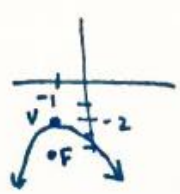
$y^2 + 4y = -8x - 12$

$y^2 + 4y + 4 = -8x - 12 + 4$

$(y+2)^2 = -8x - 8$

$\boxed{(y+2)^2 = -8(x+1)}$

vertex: $(-1, -2)$
focus: $(-1, -4)$
directrix: $y = 0$



$4p = -8$
$p = -2$

More Practice

Rewriting Conic Sections

https://www.algebra.com/algebra/homework/Quadratic-relations-and-conic-sections/Quadratic-relations-and-conic-sections.faq.question.581877.html

https://www.mathway.com/examples/algebra/conic-sections/finding-the-vertex-form-of-a-hyperbola?id=818

https://www.youtube.com/watch?v=X5rBFTVYCa0

https://www.youtube.com/watch?v=qgM37pssnWY

Homework Assignment

p.640 #49,51, p.652 #45, 47, p.664 #47,49

SAT Connection
Solution

Choice A is correct. The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. To put the equation $x^2 + y^2 + 4x - 2y = -1$ in this form, complete the square as follows:

$$x^2 + y^2 + 4x - 2y = -1$$
$$(x^2 + 4x) + (y^2 - 2y) = -1$$
$$(x^2 + 4x + 4) - 4 + (y^2 - 2y + 1) - 1 = -1$$
$$(x + 2)^2 + (y - 1)^2 - 4 - 1 = -1$$
$$(x + 2)^2 + (y - 1)^2 = 4 = 2^2$$

Therefore, the radius of the circle is 2.

Choice C is incorrect because it is the square of the radius, not the radius. Choices B and D are incorrect and may result from errors in rewriting the given equation in standard form.