8.1 Sequences

Arithmetic Sequence : $\{a_1, a_1+d, a_1+2d, \ldots\}$

explicitly - depends on n

$a_n = a_1 + (n-1)d$

recursively - depends on previous term a_{n-1}

$a_n = a_{n-1} + d, \forall n \geq 2$

where n = term #

d = common difference

a_n = n^{th} term

ex: Write an explicit and recursive formula for the arithmetic sequence whose 1^{st} term is 3 and 10^{th} term is -15.

$a_1 = 3 \quad a_{10} = -15$

$a_n = a_1 + (n-1)d$

$a_{10} = 3 + (10-1)d$

$-15 = 3 + 9d$

$-18 = 9d$

$-2 = d$

$a_n = 3 + (n-1)d$

$a_n = 3 + (n-1)(-2)$

$a_n = 3 - 2n + 2$

$\boxed{a_n = -2n + 5}$

$a_n = a_{n-1} + d$

$\boxed{a_n = a_{n-1} - 2}$

Geometric Sequence: $\{a_1, a_1r, a_1r^2, a_1r^3, \ldots\}$

explicitly

$a_n = a_1 r^{n-1}$

recursively

$a_n = a_{n-1} r \quad \forall\, n \geq 2$

ex Write an explicit and recursive formula for the geometric sequence where $a_2 = 36$ and $a_5 = -\frac{9}{2}$

$a_n = a_1 r^{n-1}$

$a_2 = a_1 r^{2-1}$

$36 = a_1 r$

$a_5 = a_1 r^{5-1}$

$-\frac{9}{2} = a_1 r^4$

$-\frac{9}{2} = a_1 r \cdot r^3$

$-\frac{9}{2} = 36 r^3$

$-\frac{9}{2} = 36 r^3$

$-\frac{1}{8} = r^3$

$-\frac{1}{2} = r$

$36 = a_1\left(-\frac{1}{2}\right)$

$-72 = a_1$

$a_n = -72\left(-\frac{1}{2}\right)^{n-1}$

$a_n = -72\left(-\frac{1}{2}\right)^{n-1}$

$a_n = -72\left(-\frac{1}{2}\right)^{n} \cdot \left(-\frac{1}{2}\right)^{-1}$

$a_n = -72\left(-\frac{1}{2}\right)^{n} \cdot -2$

$\boxed{a_n = 144\left(-\frac{1}{2}\right)^{n}}$

$a_n = a_{n-1}\left(-\frac{1}{2}\right)$

$\boxed{a_n = -\frac{1}{2} a_{n-1}}$

Limit of Sequence

* If $\lim_{n \to \infty} a_n = L$, where L is a real #, then series converges to L.

* If $\lim_{n \to \infty} a_n \neq L$, where L is a real #, then series diverges.

Determine what value the series with n^{th} term converges to or state that it diverges. Check graphically.

ex: $a_n = \frac{2n}{n+3}$

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{2n}{n+3} = \frac{\infty}{\infty}$$

by L'Hôpital

$$= \lim_{n \to \infty} \frac{2}{1}$$

$$= 2$$

$\boxed{\text{series converges to 2}}$

ex: $a_n = \frac{n}{n^2+1}$

$$\lim_{n \to \infty} \frac{n}{n^2+1} = \frac{\infty}{\infty}$$

L'Hôpital

$$= \lim_{n \to \infty} \frac{1}{2n} = \frac{1}{\infty} = 0$$

$\boxed{\text{converges to 0}}$

ex: $a_n = (-1)^n\left(\frac{n+1}{n^2+2}\right)$

$$\lim_{n\to\infty} (-1)^n\left(\frac{n+1}{n^2+2}\right)$$

$$\lim_{n\to\infty} (-1)\left(\frac{n+1}{n^2+2}\right)$$ if n is odd

$$= \lim_{n\to\infty} \frac{-n-1}{n^2+2}$$

$$= \lim_{n\to\infty} \frac{-1}{2n} = 0^-$$

$$\lim_{n\to\infty} (+1)\left(\frac{n+1}{n^2+2}\right)$$ if n is even

$$= \lim_{n\to\infty} \frac{n+1}{n^2+2}$$

$$= \lim_{n\to\infty} \frac{1}{2n} = 0^+$$

= , so series converges to 0