DATE: ____________________

L'Hôpital's Rule

Evaluate each limit.

Recall limits from way back

1. $\lim_{x\to 3} \frac{x^2-9}{x-3}$ $= \frac{3^2-9}{3-3} = \frac{0}{0}$ indeterminate form do algebra

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$

$= \lim_{x\to 3} (x+3)$

$= 3+3$

$= \boxed{6}$

2. $\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2}$ $= \frac{\sqrt{2+2}-2}{2-2} = \frac{0}{0}$ do algebra Conjugate!

$= \lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} \cdot \frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}$

$= \lim_{x\to 2} \frac{x+2-4}{(x-2)(\sqrt{x+2}+2)}$

$= \lim_{x\to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{x+2}+2)}$

$= \lim_{x\to 2} \frac{1}{\sqrt{x+2}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$

3. $\lim_{x\to \infty} \frac{x^2-9}{x-3}$ $= \frac{\infty}{\infty}$ indeterminate form

$= \infty$ b/c degree of numerator > degree of denominator

4. $\lim_{x\to \infty} \frac{x-3}{x^2-9}$ $= \frac{\infty}{\infty}$

$= 0$ b/c degree of numerator < degree of denominator

L'Hôpital's Rule

If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$

<u>and</u> $f(x)$ and $g(x)$ are differentiable and where $g'(a) \neq 0$,

then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$.

What does this mean? $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x) \to 0}{\lim_{x\to a} g(x) \to 0}$

$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f'(x)}{\lim_{x\to a} g'(x)}$ → not zero? :) → not zero? :) no more indeterminate form!

→ but $g'(a) \neq 0$

Example 1:

$\lim_{x\to 3}\frac{x^2-9}{x-3}$ $\to 3^2-9 \to \frac{0}{0}$; $3-3 \to 0$

L'Hopital's rule applies! no algebra

$= \lim_{x\to 3} \frac{2x}{1}$

$= \boxed{6}$

Example 2:

$\lim_{x\to 2}\frac{\sqrt{x+2}-2}{x-2}$ $\to \frac{0}{0}$

L'Hôpital's Rule applies!

$= \lim_{x\to 2} \frac{(x+2)^{1/2}-2}{x-2}$

$= \lim_{x\to 2} \frac{\frac{1}{2}(x+2)^{-1/2}}{1} = \lim_{x\to 2} \frac{1}{2\sqrt{x+2}} = \frac{1}{2\sqrt{2+2}} = \frac{1}{2\cdot 2} = \boxed{\frac{1}{4}}$

Example 3:

$\lim_{x\to\infty}\frac{x^2-9}{x-3}$ $\to \frac{\infty}{\infty}$

wait L'Hôpital is for $\frac{0}{0}$

but $\frac{\infty}{\infty} \to \frac{\frac{1}{0}}{\frac{1}{0}} = \frac{1}{0}\cdot\frac{0}{1} = \frac{0}{0}$ ... L'Hôpital's rule also applies for $\frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2x}{1}$

$= \boxed{\infty}$

Example 4:

$\lim_{x\to\infty}\frac{x-3}{x^2-9}$ $\to \frac{\infty}{\infty}$

L'Hôpital's Rule applies

$= \lim_{x\to\infty} \frac{1}{2x} = \boxed{0}$