Evaluate each limit. DATE: _____

1. $\lim_{x\to 0} \frac{\sin(7x)}{x}$

(A) 1

(B) 0

(C) 7

(D) ∞

(E) None of these

$\lim_{x\to 0} \frac{\sin 7x}{x} \to \frac{0}{0}$ L'Hôpital Rule applies

$= \lim_{x\to 0} \frac{7\cos 7x}{1}$

$= \frac{7(1)}{1}$

$= 7$

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2. $\lim_{x\to \pi/3} \frac{\cos(x)-\frac{1}{2}}{x-\frac{\pi}{3}}$

(A) $\frac{\sqrt{2}}{2}$

(B) $-\frac{\sqrt{3}}{2}$

(C) $-\sqrt{3}$

(D) $\frac{1}{2}$

(E) 0

$\lim_{x\to \pi/3} \frac{\cos x - \frac{1}{2}}{x-\pi/3} \to \frac{\cos \pi/3 - \frac{1}{2}}{\pi/3 - \pi/3} \to \frac{\frac{1}{2}-\frac{1}{2}}{0} \to \frac{0}{0}$ ∴ L'Hôpital Rule applies

$= \lim_{x\to \pi/3} \frac{-\sin x}{1}$

$= -\sin \pi/3$

$= -\frac{\sqrt{3}}{2}$

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3. $\lim_{x\to 3\pi} \frac{1+\tan\left(\frac{x}{4}\right)}{\cos\left(\frac{x}{2}\right)}$

(A) -2

(B) -1

(C) 1

(D) 0

(E) ∞

$\lim_{x\to 3\pi} \frac{1+\tan(\frac{x}{4})}{\cos(x/2)} \to \frac{1+\tan \frac{3\pi}{4}}{\cos 3\pi/2} \to \frac{1-1}{0} \to \frac{0}{0}$ ∴, L'Hôpital Rule applies

$= \lim_{x\to 3\pi} \frac{\frac{1}{4}\cdot \sec^2(\frac{1}{4}x)}{-\frac{1}{2}\sin(\frac{1}{2}x)}$

$= \frac{\frac{1}{4}\left(\frac{1}{\cos 3\pi/4}\right)^2}{-\frac{1}{2}\sin 3\pi/2}$

$= \frac{\frac{1}{4}(-\sqrt{2})^2}{-\frac{1}{2}(-1)}$

$= \frac{\frac{1}{4}(2)}{+\frac{1}{2}}$

$= +1$

$\frac{x}{4} = \frac{1}{4}x$ ☺

$\frac{x}{2} = \frac{1}{2}x$

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4. $\lim_{x\to 0} \frac{\tan x - x}{x^3}$

(A) 0

(B) $\frac{1}{3}$

(C) 1

(D) ∞

(E) None of these

$\lim_{x\to 0} \frac{\tan x - x}{x^3} \to \frac{0-0}{0} \to \frac{0}{0}$ ∴ L'Hôpital Rule applies

$= \lim_{x\to 0} \frac{\sec^2 x - 1}{3x^2} \to \frac{1-1}{0} \to \frac{0}{0}$ ∴, L'Hôpital again

$= \lim_{x\to 0} \frac{\sec x \tan x \cdot 2\sec x}{6x}$

$= \lim_{x\to 0} \frac{2\sec^2 x \tan x}{6x} \to \frac{2(1)(0)}{0} \to \frac{0}{0}$ ∴ L'Hôpital again !!

$= \lim_{x\to 0} \frac{\tan x(2\sec x \tan \cdot 2\sec x) + 2\sec^2 x \cdot \sec^2 x}{6}$

$= \lim_{x\to 0} \frac{4\sec^2 x \tan^2 x + 2\sec^4 x}{6} = \frac{0+2(1)}{6} = \frac{2}{6} = \frac{1}{3}$