Evaluate each limit. DATE: ____________

1. $\lim_{x\to 0} \frac{\sin(7x)}{x}$ $= \frac{0}{0}$ L'Hôpital

$= \lim_{x\to 0} \frac{\cos(7x)\cdot 7}{1}$

$= \frac{1(7)}{1}$

$= 7$

(A) 1

(B) 0

$\boxed{\text{(C) } 7}$

(D) ∞

(E) None of these

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2. $\lim_{x\to\pi/3} \frac{\cos(x)-\frac{1}{2}}{x-\frac{\pi}{3}}$ $= \frac{\cos\frac{\pi}{3}-\frac{1}{2}}{\frac{\pi}{3}-\frac{\pi}{3}} = \frac{\frac{1}{2}-\frac{1}{2}}{0} = \frac{0}{0}$ L'Hôpital

$= \lim_{x\to\pi/3} \frac{-\sin x}{1}$

$= -\sin\frac{\pi}{3}$

$= -\sqrt{3}/2$

(A) $\frac{\sqrt{2}}{2}$

$\boxed{\text{(B) } \frac{-\sqrt{3}}{2}}$

(C) $-\sqrt{3}$

(D) $\frac{1}{2}$

(E) 0

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3. $\lim_{x\to\infty} \sqrt{x^2+6x} - x$ $= \infty - \infty$

$\lim_{x\to\infty} \frac{\sqrt{x^2+6x}-x}{1} \cdot \frac{\sqrt{x^2+6x}+x}{\sqrt{x^2+6x}+x} = \lim_{x\to\infty} \frac{x^2+6x-x^2}{\sqrt{x^2+6x}+x} = \lim_{x\to\infty} \frac{6x}{\sqrt{x^2+6x}+x} = \frac{\infty}{\infty}$ so L'Hôpital

$\lim_{x\to\infty} \frac{6}{\frac{1}{2}(x^2+6x)^{-1/2}(2x+6)+1}$

$= \lim_{x\to\infty} \frac{6}{\frac{x+3}{\sqrt{x^2+6x}}+1}$

$= \lim_{x\to\infty} \frac{6\sqrt{x^2+6x}}{x+3+\sqrt{x^2+6x}}$ L'Hopital did not make problem better, so...

$\lim_{x\to\infty} \frac{6x}{\sqrt{x^2+6x}+x}$ exp N = exp D, Look @ coefficients

$= \frac{6}{1+1}$

$= 3$

$\boxed{\text{(A) } 3}$

(B) -3

(C) 6

(D) 0

(E) ∞

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4. $\lim_{x\to 0}(1+\frac{3}{x^4})^x$ $= \infty^0$

$f(x) = (1+\frac{3}{x^4})^x$

$\ln f(x) = x\ln(1+\frac{3}{x^4})$

$\lim_{x\to 0} \ln f(x) = \lim_{x\to 0} x\ln(1+\frac{3}{x^4}) = 0\cdot\infty$

$= \lim_{x\to 0} \frac{\ln(1+\frac{3}{x^4})}{\frac{1}{x}} = \frac{\infty}{\infty}$ L'Hôpital

$= \lim_{x\to 0} \frac{\frac{1}{1+3x^{-4}}\cdot -12x^{-5}}{-x^{-2}}$

$= \lim_{x\to 0} \frac{1\,(x^5)}{(1+3x^{-4})(-12)}\cdot -x^2$

$= \lim_{x\to 0} \frac{-x^7}{-12(1+\frac{3}{x^4})} = \frac{0}{-12} = 0$

$= e^{\lim_{x\to 0}(1+\frac{3}{x^4})^x}$

$= e^0$

$= \boxed{1}$

(A) 3

(B) 0

$\boxed{\text{(C) } 1}$

(D) ∞

(E) None of these