

8.2 & 8.3 Ellipses & Hyperbolas

Target 4C/4E: Investigate the geometric properties of ellipses/hyperbolas

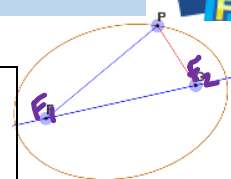
Target 4D/4F: Derive the standard equation of an ellipse/hyperbola and graph given two or three criteria

Ellipse



Definition (in your own words) <https://www.mathsisfun.com/geometry/ellipse.html>

Set of all points in a plane whose distances from 2 points have a constant sum.
(foci)
 $PF_1 + PF_2 = \text{constant}$



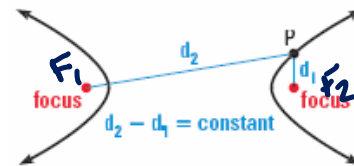
Major Axis Parallel to the x-axis	Sketch	Example
Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $a > b$	$\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$
Center	(h, k)	center: $(1, -2)$ $a^2 = 16$ $a = 4$
Vertices	$(h+a, k)$ and $(h-a, k)$	vertices: $(5, -2)$ $(-3, -2)$ $b^2 = 9$ $b = 3$
Foci	$(h+c, k)$ and $(h-c, k)$	foci: $(1+\sqrt{7}, -2)$ $(1-\sqrt{7}, -2)$ $16 = 9 + c^2$ $7 = c^2$ $\pm\sqrt{7} = c$
Focal Axis	$y = k$	
Semi-major Axis	a	
Semi-minor Axis	b	
Pythagorean relation	$a^2 = b^2 + c^2$	
Eccentricity	$e = \frac{c}{a}$	

Major Axis Parallel to the y-axis	Sketch	Example
Standard Form of Equation	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ $a > b$	$\frac{(x-1)^2}{25} + \frac{(y+2)^2}{36} = 1$
Center	(h, k)	center: $(1, -2)$ $a^2 = 36$ $a = 6$
Vertices	$(h, k+a)$ and $(h, k-a)$	vertices: $(1, -8)$ $(1, 4)$ $b^2 = 25$ $b = 5$
Foci	$(h, k+c)$ and $(h, k-c)$	foci: $(1, -2+\sqrt{11})$ $(1, -2-\sqrt{11})$ $36 = 25 + c^2$ $11 = c^2$ $\pm\sqrt{11} = c$
Focal Axis	$x = h$	
Semi-major Axis	a	
Semi-minor Axis	b	
Pythagorean relation	$a^2 = b^2 + c^2$	
Eccentricity	$e = \frac{c}{a}$	

Hyperbola

Definition (in your own words)

Set of all points in a plane whose distances from 2 points is a constant difference (foci)
 $PF_1 - PF_2 = \text{constant}$



Major Axis Parallel to the x-axis		Sketch	Example
Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$a > b$	$\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$ $\frac{(x-1)^2}{4^2} - \frac{(y+2)^2}{3^2} = 1$
Center	(h, k)		Center: $(1, -2)$
Vertices	$(h+a, k)$ and $(h-a, k)$		Vertices: $(5, -2)$ and $(-3, -2)$
Foci	$(h+c, k)$ and $(h-c, k)$		Foci: $(6, -2)$ and $(-4, -2)$
Semi-transverse Axis	a		$a=4$
Semi-conjugate Axis	b		$c^2 = 4^2 + 3^2$
Asymptotes	$y = \pm \frac{b}{a}(x-h) + k$		$c^2 = 25$
Pythagorean relation	$c^2 = a^2 + b^2$		$c = 5$
Eccentricity	$e = \frac{c}{a}$		

Major Axis Parallel to the y-axis		Sketch	Example
Standard Form of Equation	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	$a > b$	$\frac{(y+2)^2}{9} - \frac{(x-1)^2}{25} = 1$ $\frac{(y+2)^2}{3^2} - \frac{(x-1)^2}{5^2} = 1$
Center	(h, k)		Center: $(1, -2)$
Vertices	$(h, k+a)$ and $(h, k-a)$		Vertices: $(1, 1)$ and $(1, -5)$
Foci	$(h, k+c)$ and $(h, k-c)$		Foci: $(1, 2 + \sqrt{34})$ and $(1, 2 - \sqrt{34})$
Semi-transverse Axis	a		$a=3$
Semi-conjugate Axis	b		$c^2 = 25 + 9$
Asymptotes	$y = \pm \frac{a}{b}(x-h) + k$		$c^2 = 34$
Pythagorean relation	$c^2 = a^2 + b^2$		$c = \sqrt{34}$
Eccentricity	$e = \frac{c}{a}$		

Unit 4 (Chapter 8): Conic Sections

Pre-Calculus 2016-2017

Examples:

1. Sketch the graph of: $\frac{(x+2)^2}{16} + \frac{(y+1)^2}{25} = 1$

Label the center, vertices, and foci

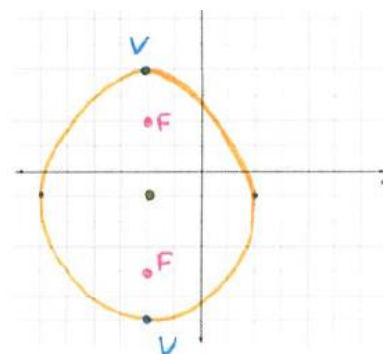
$$\frac{(x-(-2))^2}{4^2} + \frac{(y-(-1))^2}{5^2} = 1$$

Center: $(-2, -1)$ $a=5$

Vertices: $(-2, 4)$ and $(-2, -6)$

Foci: $(-2, 2)$ and $(-2, -4)$

$$\begin{aligned} 25 &= 16 + c^2 \\ 9 &= c^2 \\ 3 &= c \end{aligned}$$



2. Sketch the graph of: $\frac{x^2}{4} - \frac{(y+1)^2}{21} = 1$

Label the center, vertices, and foci

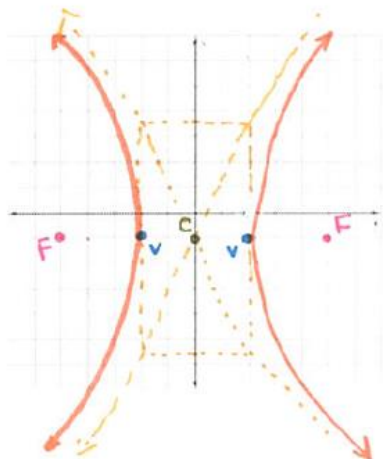
$$\frac{(x-0)^2}{2^2} - \frac{(y-(-1))^2}{(\sqrt{21})^2} = 1$$

Center: $(0, -1)$ $a=2$

Vertices: $(2, -1)$ and $(-2, -1)$

Foci: $(5, -1)$ and $(-5, -1)$

$$\begin{aligned} c^2 &= 4 + 21 \\ c^2 &= 25 \\ c &= 5 \end{aligned}$$



3. Identify the type of conic and find the center, vertices, and foci: $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{4} = 1$

Ellipse $\frac{(x-(-3))^2}{4^2} + \frac{(y-1)^2}{2^2} = 1$

Center: $(-3, 1)$ $a=4$

Vertices: $(1, 1)$ and $(-7, 1)$

Foci: $(-3+2\sqrt{3}, 1)$ and $(-3-2\sqrt{3}, 1)$

$$\begin{aligned} 16 &= c^2 + 4 \\ 12 &= c^2 \\ \sqrt{12} &= c \\ 2\sqrt{3} &= c \text{ or} \end{aligned}$$

4. Identify the type of conic and find the center, vertices, and foci: $\frac{(y+2)^2}{9} - \frac{(x-6)^2}{16} = 1$

Hyperbola $\frac{(y-(-2))^2}{3^2} - \frac{(x-6)^2}{4^2} = 1$

Center: $(6, -2)$ $a=3$

Vertices: $(6, 1)$ and $(6, -5)$

Foci: $(6, 3)$ and $(6, -7)$

$$\begin{aligned} c^2 &= 9 + 16 \\ c^2 &= 25 \\ c &= 5 \end{aligned}$$

More Practice**Ellipses**

https://www.youtube.com/watch?v=3O_TMiP9piI

<https://www.khanacademy.org/math/algebra-home/alg-conic-sections/alg-center-and-radii-of-an-ellipse/v/conic-sections-intro-to-ellipses>

<http://www.mathwarehouse.com/ellipse/equation-of-ellipse.php>

Hyperbolas

http://www.algebralab.org/lessons/lesson.aspx?file=algebra_conics_hyperbola.xml

<http://www.purplemath.com/modules/hyperbola2.htm>

<http://www.ck12.org/book/CK-12-Algebra-II-with-Trigonometry-Concepts/section/10.9/>

<http://www.mathwarehouse.com/hyperbola/graph-equation-of-a-hyperbola.php>

<https://www.youtube.com/watch?v=FcAyzSMd0iU>

https://www.youtube.com/watch?v=Z6cwpsDC_5A

<https://www.youtube.com/watch?v=i6vM82SNAUk>

Homework Assignment

p. 639 #4,5, p.652 #7-10all, 37,39, p.663 #7-10all,39,4