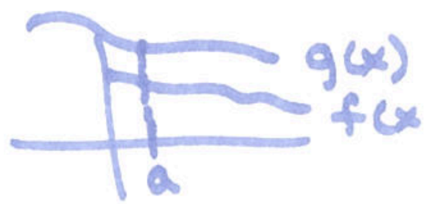
Comparison Test

$f(x)$ and $g(x)$ cont on $[a, \infty)$ and

$$0 \leq f(x) \leq g(x) \quad \forall x \geq a$$

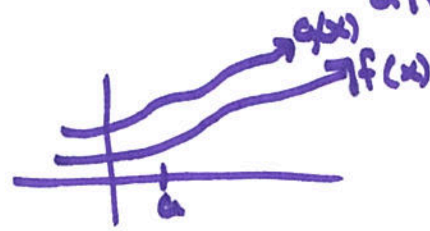
① If $\int_a^\infty g(x)\,dx$ converges, then $\int_a^\infty f(x)\,dx$ also converges.

If $g(x) \geq f(x)$ and $g(x)$ converges, then $f(x)$ also will converge



② If $\int_a^\infty f(x)\,dx$ diverges, then $\int_a^\infty g(x)\,dx$ also diverges

If $f(x) \leq g(x)$ and $f(x)$ diverges, then $g(x)$ also will diverge



ex: $\displaystyle\int_1^\infty \frac{dx}{x^2+3}$

Since $\frac{1}{x^2} \geq \frac{1}{x^2+3} \geq 0$,

and $\displaystyle\int_1^\infty \frac{1}{x^2}dx = \lim_{a\to\infty}\int_1^a \frac{1}{x^2}dx$

$= \lim_{a\to\infty}\left(-x^{-1}\right)\Big|_1^a$

$= \lim_{a\to\infty}\left(-\frac{1}{a}+1\right)$

$= 1$

converges

then $\displaystyle\int_1^\infty \frac{1}{x^2+3}dx$ also $\boxed{\text{converges}}$

ex: $\displaystyle\int_1^\infty \frac{2x+1}{(x+1)^2}dx$

$\frac{2x+1}{(x+1)^2} \geq \frac{2x}{x^2} \geq \frac{2}{x}$

Since $0 \leq \frac{2}{x} \leq \frac{2x+1}{(x+1)^2}$

and $\displaystyle\int_1^\infty \frac{2}{x}dx = \lim_{a\to\infty}\int_1^a \frac{2}{x}dx$

$= \lim_{a\to\infty}\left(2\ln|x|\right)\Big|_1^a$

$= \lim_{a\to\infty}\left(2\ln a - 2\ln 1\right)$

$= \infty$ diverges.

then $\displaystyle\int_1^\infty \frac{2x+1}{(x+1)^2}dx$ also $\boxed{\text{diverges}}$