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Improper Integrals Practice (Relay Style)

Evaluate the integral.

1. $\int_1^{\infty} \frac{1}{(4x+1)^2}\,dx$

$\int_1^{\infty} \frac{1}{(4x+1)^2}dx = \lim_{a\to\infty} \int_1^a (4x+1)^{-2}dx$

$= \lim_{a\to\infty} \left(-\frac{1}{4}(4x+1)^{-1}\right)\Big|_1^a$

$= \lim_{a\to\infty} \left(-\frac{1}{4(4a+1)} + \frac{1}{4(5)}\right)$

$= \boxed{\frac{1}{20}}$

$u = 4x+1$
$du = 4\,dx$
$\frac{1}{4}du = dx$
$\frac{1}{4}\int u^{-2}du$
$-\frac{1}{4}u^{-1}$
$-\frac{1}{4}(4x+1)^{-1}$

Evaluate the integral.

2. If #1 diverges, replace ANS with 2.

$\int_0^{80(ANS)} \frac{1}{\sqrt{x}}\,dx$

$\int_0^{80(\frac{1}{20})} \frac{1}{\sqrt{x}}dx = \int_0^4 \frac{1}{\sqrt{x}}dx = \lim_{a\to 0^+} \int_a^4 x^{-1/2}dx$

$= \lim_{a\to 0^+} 2x^{1/2}\Big|_a^4$

$= \lim_{a\to 0^+} (2\sqrt{4} - 2\sqrt{a})$

$= \boxed{4}$

Evaluate the integral.

3. If #2 diverges, replace ANS with 1.

$\int_{ANS/2}^{\infty} \frac{x}{x^2+1}\,dx$

$\int_{4/2}^{\infty} \frac{x}{x^2+1}dx = \lim_{a\to\infty} \int_2^a \frac{x}{x^2+1}dx$

$= \lim_{a\to\infty} \frac{1}{2}\ln|x^2+1|\Big|_2^a$

$= \lim_{a\to\infty} \left(\frac{1}{2}\ln(a^2+1) - \frac{1}{2}\ln 5\right)$

$= \infty$

$\therefore \int_2^{\infty} \frac{x}{x^2+1}dx$ diverges.

$u = x^2+1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$
$\frac{1}{2}\int \frac{1}{u}du$
$\frac{1}{2}\ln|u|$
$\frac{1}{2}\ln|x^2+1|$

Evaluate the integral.

4. If #3 diverges, replace ANS with $\ln 2$.

$\int_{ANS}^{\infty} xe^{-x}\,dx$

	u	dv
+	x	e^{-x}
−	1	$-e^{-x}$
	0	e^{-x}

$\int_{\ln 2}^{\infty} xe^{-x}dx = \lim_{a\to\infty} \int_{\ln 2}^a xe^{-x}dx$

$= \lim_{a\to\infty} (-xe^{-x} - e^{-x})\Big|_{\ln 2}^a$

$= \lim_{a\to\infty} \left(-ae^{-a} - e^{-a} + \ln 2\, e^{-\ln 2} - e^{-\ln 2}\right)$

$= 0 - 0 + \frac{\ln 2}{e^{\ln 2}} - \frac{1}{e^{\ln 2}} = \boxed{\frac{\ln 2}{2} - \frac{1}{2}}$

$\lim_{a\to\infty} ae^{-a} \to \infty \cdot 0$

$\lim_{a\to\infty} \frac{a}{e^a} \to \frac{\infty}{\infty}$

$\therefore$, By L'Hôpital,

$= \lim_{a\to\infty} \frac{1}{e^a}$

$= 0$

Determine if the integral converges or diverges.

5. If #4 diverges, replace ANS with 0.

$$\int_{2(ANS)-\ln 2}^{\infty} \frac{x+1}{x^2+x+1}\,dx$$

$2(\ln 2 - \frac{1}{2}) - \ln 2$

$\int_1^\infty \frac{x+1}{x^2+x+1}dx$

$\frac{x+1}{x^2+x+1}$ compares to $\frac{x}{x^2+x} = \frac{x}{x(x+1)} = \frac{1}{x+1}$

$0 \le \frac{1}{x+1} \le \frac{x+1}{x^2+x+1}$ we want $\int \frac{1}{x+1}dx$ to diverge

$\int_1^\infty \frac{1}{x+1}dx = \lim_{a\to\infty} \int_1^a \frac{1}{x+1}dx$

$= \lim_{a\to\infty} \ln|x+1| \Big|_1^a$

$= \lim_{a\to\infty} (\ln|a+1| + \ln 2)$

$= \infty$

Since $0 \le \frac{1}{x+1} \le \frac{x+1}{x^2+x+1}$ and $\int_1^\infty \frac{1}{x+1}dx$ diverges, then $\int_1^\infty \frac{x+1}{x^2+x+1}dx$ diverges by comparison test.

OR $\frac{x+1}{x^2+x+1}$ compares to $\frac{x}{x^2} = \frac{1}{x}$, but $\frac{x+1}{x^2+x+1} \le \frac{1}{x}$ and $\int \frac{1}{x}dx$ diverges

So, compare to $\frac{1}{3x}$

$0 \le \frac{1}{3x} \le \frac{x+1}{x^2+x+1}$, $\int_1^\infty \frac{1}{3x}dx = \lim_{a\to\infty} \frac{1}{3}\int_1^a \frac{1}{x}dx$

$= \lim_{a\to\infty} \frac{1}{3}\ln x \Big|_1^a$

$= \lim_{a\to\infty} (\frac{1}{3}\ln a - 0)$

$= \infty$

Since $\int_1^\infty \frac{1}{3x}dx$ diverges and $0 \le \frac{1}{3x} \le \frac{x+1}{x^2+x+1}$, then $\int_1^\infty \frac{x+1}{x^2+x+1}dx$ diverges by Comparison test

Determine if the integral converges or diverges.

6. If #5 diverges, replace ANS with 1.
If #5 converges, replace ANS with 0.

$$\int_{ANS}^{\infty} \frac{dx}{\sqrt{x^3+x+1}}$$

$\int_1^\infty \frac{dx}{\sqrt{x^3+x+1}}$

$\frac{1}{\sqrt{x^3+x+1}}$ compares to $\frac{1}{\sqrt{x^3}}$

$0 \le \frac{1}{\sqrt{x^3+x+1}} \le \frac{1}{\sqrt{x^3}}$ we want $\int \frac{1}{\sqrt{x^3}}dx$ to converge

$\int_1^\infty \frac{1}{\sqrt{x^3}}dx = \lim_{a\to\infty} \int_1^a x^{-3/2}dx$

$= \lim_{a\to\infty} (-2x^{-1/2})\Big|_1^a$

$= \lim_{a\to\infty} (\frac{-2}{\sqrt{a}} + 2)$

$= 2$

Since $\int_1^\infty \frac{1}{\sqrt{x^3}}dx$ converges and $0 \le \frac{1}{\sqrt{x^3+x+1}} \le \frac{1}{\sqrt{x^3}}$, then $\int_1^\infty \frac{1}{\sqrt{x^3+x+1}}dx$ converges by comparison test.

Determine if the integral converges or diverges.

7. If #6 diverges, replace ANS with 0.
If #6 converges, replace ANS with 1.

$$\int_{ANS}^{\infty} \frac{dx}{x^2+x+1}$$

$\int_1^\infty \frac{dx}{x^2+x+1}$

$\frac{1}{x^2+x+1}$ compares to $\frac{1}{x^2}$

$0 \le \frac{1}{x^2+x+1} \le \frac{1}{x^2}$ we want $\int \frac{1}{x^2}dx$ to converge

$\int_1^\infty \frac{1}{x^2}dx = \lim_{a\to\infty} \int_1^a x^{-2}dx$

$= \lim_{a\to\infty} (-x^{-1})\Big|_1^a$

$= \lim_{a\to\infty} (-\frac{1}{a} + 1)$

$= 1$

Since $\int_1^\infty \frac{1}{x^2}dx$ converges and $0 \le \frac{1}{x^2+x+1} \le \frac{1}{x^2}$, then $\int_1^\infty \frac{1}{x^2+x+1}dx$ converges by comparison test.

Evaluate the integral.

8. If #7 diverges, replace ANS with 2.
If #7 converges, replace ANS with 1.

$$\int_0^{ANS} \frac{dx}{x^{1/3}}$$

$\int_0^1 \frac{dx}{x^{1/3}} = \int_0^1 x^{-1/3}dx = \lim_{a\to 0^+} \int_a^1 x^{-1/3}dx$

$= \lim_{a\to 0^+} (\frac{3}{2}x^{2/3})\Big|_a^1$

$= \lim_{a\to 0^+} (\frac{3}{2} - \frac{3}{2}a^{3/2})$

$= \boxed{\frac{3}{2}}$

ANSWERS:

1. $\int_1^\infty \frac{1}{(4x+1)^2} dx = \frac{1}{20}$
2. $\int_0^4 \frac{1}{\sqrt{x}} dx = 4$
3. $\int_2^\infty \frac{x}{x^2+1} dx$ divergent
4. $\int_{\ln 2}^\infty xe^{-x}\, dx = \frac{1+\ln 2}{2}$
5. $\int_1^\infty \frac{x+1}{x^2+x+1} dx$ divergent by comparison test
6. $\int_1^\infty \frac{dx}{\sqrt{x^3+x+1}}$ converges by comparison test
7. $\int_1^\infty \frac{dx}{x^2+x+1}$ converges by comparison test
8. $\int_0^{ANS} \frac{dx}{x^{1/3}} = \frac{3}{2}$