DATE: ________________

Improper Integrals Practice (Relay Style)

Evaluate the integral.

1. $\int_1^\infty \frac{1}{(4x+1)^2}\,dx$

$\lim_{a\to\infty} \int_1^a (4x+1)^{-2}\,dx$

$\lim_{a\to\infty} -\frac{1}{4}(4x+1)^{-1}\Big|_1^a$

$\lim_{a\to\infty} -\frac{1}{4}\left(\frac{1}{4a+1} - \frac{1}{5}\right)$

$-\frac{1}{4}\left(-\frac{1}{5}\right) = \boxed{\frac{1}{20}}$ ANS

Evaluate the integral.

2. If #1 diverges, replace ANS with 2.

$\int_0^{80(ANS)} \frac{1}{\sqrt{x}}\,dx$

$\int_0^4 \frac{1}{\sqrt{x}}\,dx$

$\lim_{a\to 0} \int_a^4 x^{-1/2}\,dx$

$\lim_{a\to 0} (2x^{1/2})\Big|_a^4$

$\lim_{a\to 0} (2(2) - 2\sqrt{a})$

$\boxed{4}$ ANS

Evaluate the integral.

3. If #2 diverges, replace ANS with 1.

$\int_{ANS/2}^\infty \frac{x}{x^2+1}\,dx$

$\int_2^\infty \frac{x}{x^2+1}\,dx$

$\lim_{a\to\infty} \int_2^a \frac{x}{x^2+1}\,dx$ $\quad u = x^2+1 \quad u(a) = a^2+1$ $\quad \frac{du}{2x} = dx \quad u(2) = 5$

$\lim_{a\to\infty} \frac{1}{2}\int_5^{a^2+1} u^{-1}\,du$

$\lim_{a\to\infty} \frac{1}{2}\ln|u|\Big|_5^{a^2+1} = \lim_{a\to\infty} \frac{1}{2}(\ln(a^2+1) - \ln 5)$

DIVERGES

Evaluate the integral.

4. If #3 diverges, replace ANS with ln 2.

$\int_{ANS}^\infty xe^{-x}\,dx = \int_{\ln 2}^\infty xe^{-x}\,dx$

$\lim_{a\to\infty} \int_{\ln 2}^a xe^{-x}\,dx$ $\quad u = x \quad du = dx \quad \int dv = \int e^{-x}dx \quad v = -e^{-x}$

$\lim_{a\to\infty} \left(-xe^{-x}\Big|_{\ln 2}^a + \int_{\ln 2}^a e^{-x}\,dx\right)$

$\lim_{a\to\infty} \left(-ae^{-a} + \ln 2\, e^{-\ln 2} + (-e^{-x})\Big|_{\ln 2}^a\right)$

$\lim_{a\to\infty} \left(-\frac{a}{e^a} + \frac{1}{2}\ln 2 - e^{-a} + e^{-\ln 2}\right)$

$\lim_{a\to\infty} \left(-\frac{a}{e^a}\right) + \lim_{a\to\infty} \left(\frac{1}{2}\ln 2 - \frac{1}{e^a} + \frac{1}{2}\right)$

$\lim_{a\to\infty} \left(-\frac{1}{e^a}\right) + \lim_{a\to\infty} \left(\frac{1}{2}\ln 2 - \frac{1}{e^a} + \frac{1}{2}\right) = \boxed{\frac{1}{2}\ln 2 + \frac{1}{2}}$ ← ANS

Determine if the integral converges or diverges.

5. If #4 diverges, replace ANS with 0.

$\int_{2(ANS)-\ln 2}^\infty \frac{x+1}{x^2+x+1}\,dx$

$\int_1^\infty \frac{x+1}{x^2+x+1}\,dx$ $\qquad \frac{1}{x} \le \frac{x}{3x^2} \le \frac{x+1}{x^2+x^2+x^2} \le \frac{x+1}{x^2+x+1}$

Since $\int_1^\infty \frac{1}{x}\,dx$ diverges, by comparison test,

$\int_1^\infty \frac{x+1}{x^2+x+1}\,dx$ also diverges

Determine if the integral converges or diverges.

6. If #5 diverges, replace ANS with 1.
If #5 converges, replace ANS with 0.

$\int_{ANS}^\infty \frac{dx}{\sqrt{x^3+x+1}}$

$\int_1^\infty \frac{dx}{\sqrt{x^3+x+1}}$ $\qquad$ by comparison test $\quad x^{3/2} \le \sqrt{x^3+x+1}$ $\quad$ so $\frac{1}{\sqrt{x^3+x+1}} \le \frac{1}{x^{3/2}}$

$\int_1^\infty x^{-3/2}\,dx = \lim_{a\to\infty} \int_1^a x^{-3/2}\,dx$

$= \lim_{a\to\infty} (-2x^{-3/2})\Big|_1^a$

$= \lim_{a\to\infty} (-2a^{-3/2} + 2) \Rightarrow$ converges

so $\int_1^\infty \frac{dx}{\sqrt{x^3+x+1}}$ also converges

Determine if the integral converges or diverges.

7. If #6 diverges, replace ANS with 0.
If #6 converges, replace ANS with 1.

$\int_{ANS}^\infty \frac{dx}{x^2+x+1}$

$\int_1^\infty \frac{dx}{x^2+x+1}$ $\qquad$ comparison test $\quad x^2 \le x^2+x+1 \quad \frac{1}{x^2+x+1} \le \frac{1}{x^2}$

$\lim_{a\to\infty} \int_1^a \frac{1}{x^2}\,dx = \lim_{a\to\infty} (-x^{-1})\Big|_1^a$

$= \lim_{a\to\infty} \left(-\frac{1}{a} + 1\right) = 1$ converges

so $\int_1^\infty \frac{dx}{x^2+x+1}$ also converges

Evaluate the integral.

8. If #7 diverges, replace ANS with 2.
If #7 converges, replace ANS with 1.

$\int_0^{ANS} \frac{dx}{x^{1/3}}$

$\int_0^1 x^{-1/3}\,dx = \frac{3}{2}x^{2/3}\Big|_0^1$

$= \boxed{\frac{3}{2}}$