DATE: _______________

Power Series

Infinite Series

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{k=1}^{\infty} a_k$$

Finding the sum of an infinite Series by finding the Partial Sums

If a partial sum converges to S, then the series converges to S.

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3$$
$$\vdots$$
$$S_n = \sum_{k=1}^{\infty} a_k$$

Example: Does the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots$ converge?

✱If the infinite series converges, then $\lim_{k\to\infty} a_k = 0$.

(The converse is not necessarily true, but the contrapositive is true.)

Converse:

If ___________________, then _______________________________________. (not necessarily true)

Contrapositive:

✱ If ______________________, then ____. (is true)

nth term test

Geometric Series

$$a + ar + ar^2 + \cdots + ar^{n-1} + \cdots = \sum_{n=1}^{\infty} ar^{n-1}$$

converges to :   , if

diverges if:

Power Series

Centered at $x = 0$,

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots + c_n x^n + \cdots$$

Example:

$$\sum_{n=1}^{\infty} \frac{x^n}{n}$$

Centered at $x = a$,

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots + c_n(x-a)^n + \cdots$$

Example:

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$$

Example 1:
Write the Geometric Series below as a function.

$$\sum_{n=0}^{\infty} x^n$$

Example 2:
Express the function as a power series and find the interval of convergence.

$$f(x) = \frac{1}{1+x}$$

Example 3:
Express the function as a power series and find the interval of convergence.

$$g(x) = \frac{1}{1+x^2}$$

Example 4:
Express the function as a power series and find the interval of convergence.

$h(x) = \frac{1}{5+x}$

Example 5:
Express the function as a power series and find the interval of convergence.

$p(x) = \frac{x^5}{3-6x^3}$