Permutations

Permutations.tns

DATE ____________

Problem 1 – An introduction

A password must contain 5 unique lowercase letters. How many possible passwords are there?

A. 3,125 **B.** 100,000 $\boxed{\textbf{C.}}$ 7,893,600 **D.** 11,881,376

- Explain why you chose the answer you did.

put letters in particular places

$$ {}_{26}P_5 = \frac{26!}{(26-5)!} = \frac{26\cdot 25\cdot 24\cdot 23\cdot 22\cdot \cancel{21!}}{\cancel{21!}} = 7,893,600 $$

Problem 2 – Factorials and the Fundamental Counting Principle

- Evaluate the following. $5\cdot 4\cdot 3\cdot 2\cdot 1 =$ 120

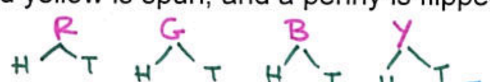
  $5! =$ 120

  $0! =$ 1

  $(5-2)! =$ $3! = 6$

  $5! - 2! =$ $120 - 2 = 118$

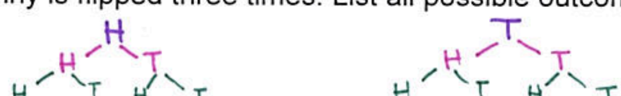
- A spinner with four equal sections colored red, green, blue, and yellow is spun, and a penny is flipped. List all possible outcomes.



$4(2) = 8$ possible outcomes

RH, RT, GH, GT, BH, BT, YH, YT

- A penny is flipped three times. List all possible outcomes.



$2(2)(2) = 8$ possible outcomes

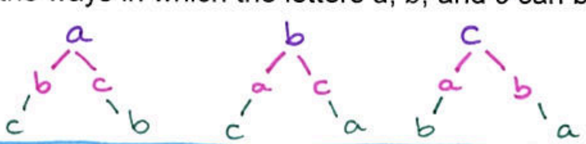
HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

- State the Fundamental Counting Principle in your own words.

Sample answer: a choices in b ways is $a\cdot b$

Problem 3 – *n* objects taken *n* at a time

- List all the ways in which the letters *a*, *b*, and *c* can be arranged.



abc, acb, bac, bca, cab, cba

- What multiplication expression can be used to find the answer? $3\cdot 2\cdot 1 = 3!$
- Complete this equation: ${}_nP_n = \boxed{n!}$ permutation of n items taken n at a time ☺
- Find how many different ways you can arrange the letters in the word **NUMBER**. $\underbrace{\text{NUMBER}}_{\text{6 letters}}$ ${}_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6!$ $6! = 720$

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Problem 4 – *n* objects taken *r* at a time

- List all of the ways to arrange *two* of the following 4 letters: *a*, *b*, *c*, and *d*.

a: b c d   b: a c d   c: a b d   d: a b c

$\boxed{ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc}$ → 12 ways

- What multiplication expression can be used to find the answer? $4 \cdot 3$

notice: this equals $\frac{4\cdot3\cdot2\cdot1}{2\cdot1} = \frac{4!}{2!}$

- Complete this equation: $_nP_r = \frac{\boxed{n!}}{\boxed{(n-r)!}}$ #of permutations of n objects taken r at a time... ☺

- A collector has 16 statues. In how many ways can the collector arrange 5 of the statues on a shelf? $_{16}P_5 = \frac{16!}{(16-5)!} = \frac{16!}{11!} = \boxed{524{,}160}$

Problem 5 – Practice

- A certain password must contain 5 unique lowercase letters. How many possible passwords are there? $_{26}P_5 = \frac{26!}{(26-5)!} = \boxed{7893600}$

- Use permutations to find the number of ways the letters in the word **FLOWER** can be arranged. $_6P_6 = \boxed{720}$

- Ten people are in a race. Use permutations to find the number of ways 1st, 2nd, and 3rd places can be awarded. $_{10}P_3 = \boxed{720}$

- **CHALLENGE:** A password must have 3 unique lowercase letters and 5 unique digits. Find the number of possible passwords if the letters must stay grouped together and the digits must stay grouped together. 1st lowercase, then digits + 1st digits, then lowercase

$(_{26}P_3)(_{10}P_5) + (_{10}P_5)(_{26}P_3)$

$2(_{26}P_3)(_{10}P_5) =$ → 94,348,800

Extension

Read p.705.
Find the number of distinguishable permutations of the letters in each of these words.

- **PIZZA** 2 Z's $\frac{5!}{2!} = \boxed{60}$
- **SUCCESS** $\frac{7!}{3!2!} = \boxed{420}$
  2 C's
  3 S's
- **COOKBOOK** $\frac{8!}{4!2!} = \boxed{840}$
  4 O's, 2 K's
- **MISSISSIPPI** $\frac{11!}{4!4!2!} = \boxed{34{,}650}$
  4 S's
  4 I's
  2 P's