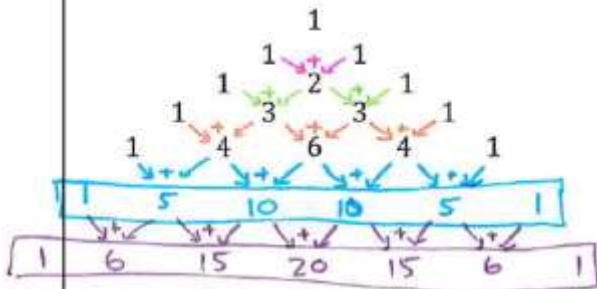


9.2 The Binomial Theorem

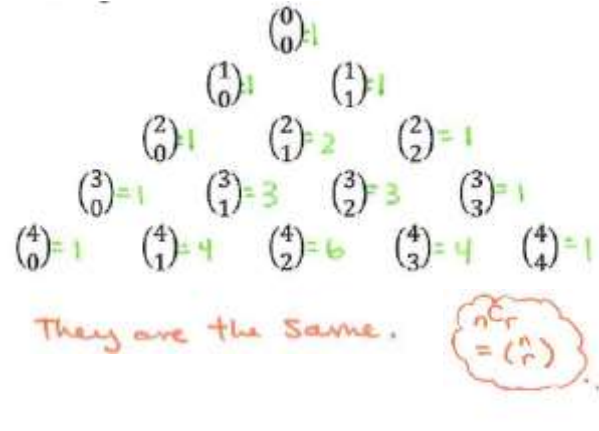
Target 7A: Expand the power of a binomial using the Binomial Theorem

Review of Prior Concepts

1. Predict the next 2 rows:



2. How does the following compare with problem #1?



More Practice

Pascal's Triangle

<http://www.mathsisfun.com/pascals-triangle.html><https://youtu.be/XMriWTvPXHI>

SAT Connection

Heart of Algebra

4. Create an equivalent form of an algebraic expression

Example:

$$9a^4 + 12a^2b^2 + 4b^4$$

Which of the following is equivalent to the expression shown above?

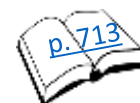
A) $(3a^2 + 2b^2)^2$

B) $(3a + 2b)^4$

C) $(9a^2 + 4b^2)^2$

D) $(9a + 4b)^4$

[Solution](#)



Find the terms in:

$$(a + b)^0 =$$

$$(a + b)^1 =$$

$$(a + b)^2 =$$

$$(a + b)^3 =$$

$$(a + b)^4 =$$

$$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$\rightarrow \binom{5}{0}a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \binom{5}{4}ab^4 + \binom{5}{5}b^5$
 coefficient

Binomial Coefficient: $\binom{n}{r}$ or nCr

Binomial Theorem

$$(a + b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n-1}a^1b^{n-1} + \binom{n}{n}a^0b^n$$

$$= \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

Example 1: Expand $(x + 2)^6$

\rightarrow always add to = n

$$= \binom{6}{0}x^6(2)^0 + \binom{6}{1}x^5(2)^1 + \binom{6}{2}x^4(2)^2 + \binom{6}{3}x^3(2)^3 + \binom{6}{4}x^2(2)^4 + \binom{6}{5}x(2)^5 + \binom{6}{6}x^0(2)^6$$

$$= x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$$

Example 2: Expand $(x - 3)^4$

$$= \binom{4}{0}x^4(-3)^0 + \binom{4}{1}x^3(-3)^1 + \binom{4}{2}x^2(-3)^2 + \binom{4}{3}x^1(-3)^3 + \binom{4}{4}x^0(-3)^4$$

$$= x^4 - 12x^3 + 54x^2 - 108x + 81$$

Finding one Term in a Binomial Expansion

$$(a + b)^n = \underbrace{\binom{n}{0}a^n b^0}_{1^{st} \text{ term}} + \underbrace{\binom{n}{1}a^{n-1}b^1}_{2^{nd} \text{ term}} + \underbrace{\binom{n}{2}a^{n-2}b^2}_{3^{rd} \text{ term}} + \dots + \underbrace{\binom{n}{n-1}a^1b^{n-1} + \binom{n}{n}a^0b^n}_{n^{th} \text{ term}}$$

In general, the r^{th} term is: $\binom{n}{r-1} a^{n-(r-1)} b^{r-1}$
 or $\binom{n}{r-1} a^{n-r+1} b^{r-1}$

Example 3: Find the coefficient of the 8th term of $(x + 2)^{11}$

$$\binom{11}{7} x^{11-7} (2)^7$$

$$= 330 x^4 \cdot 128 = 42240 x^4$$

coefficient is: 42240

Example 4: Find the coefficient of the 3rd term of $(x - 3)^6$

$$\begin{aligned} & \binom{6}{2} x^{6-2} (-3)^2 \\ & = 15x^4 \cdot 9 = 135x^4 \\ & \text{coefficient is: } \boxed{135} \end{aligned}$$

Example 5: Find the 4th term of $(x^2 + y)^5$

$$\begin{aligned} & \binom{5}{3} (x^2)^{5-3} (y)^3 \\ & = 10(x^2)^2 y^3 \\ & = \boxed{10x^4 y^3} \end{aligned}$$

More Practice

Binomial Theorem

<http://www.purplemath.com/modules/binomial.htm>

<https://www.mathsisfun.com/algebra/binomial-theorem.html>

<https://people.richland.edu/james/lecture/m116/sequences/binomial.html>

<https://youtu.be/ojFuf9RYmzI>

Homework Assignment

p.717 #1–15odd,27,28

SAT Connection

Solution

Choice A is correct. If a polynomial expression is in the form $(x)^2 + 2(x)(y) + (y)^2$, then it is equivalent to $(x + y)^2$. Because $9a^4 + 12a^2b^2 + 4b^4 = (3a^2)^2 + 2(3a^2)(2b^2) + (2b^2)^2$, it can be rewritten as $(3a^2 + 2b^2)^2$.

Choice B is incorrect. The expression $(3a + 2b)^4$ is equivalent to the product $(3a + 2b)(3a + 2b)(3a + 2b)(3a + 2b)$. This product will contain the term $4(3a)^3(2b) = 216a^3b$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $216a^3b$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (3a + 2b)^4$.

Choice C is incorrect. The expression $(9a^2 + 4b^2)^2$ is equivalent to the product $(9a^2 + 4b^2)(9a^2 + 4b^2)$. This product will contain the term $(9a^2)(9a^2) = 81a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $81a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a^2 + 4b^2)^2$.

Choice D is incorrect. The expression $(9a + 4b)^4$ is equivalent to the product $(9a + 4b)(9a + 4b)(9a + 4b)(9a + 4b)$. This product will contain the term $(9a)(9a)(9a)(9a) = 6,561a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $6,561a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a + 4b)^4$.