DATE: __________

9.2 Taylor Series

Recall the Power Series:

$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots + c_n(x-a)^n + \cdots$

for $x = a$, $f(a) =$

so, $c_0 =$

$f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + 4c_4(x-a)^3 + \cdots + nc_n(x-a)^{n-1} + \cdots$

then, $f'(a) =$

so, $c_1 =$

$f''(x) = 2c_2 + 2 \cdot 3c_3(x-a) + 3 \cdot 4c_4(x-a)^2 + \cdots + (n-1)n \cdot c_n(x-a)^{n-2} + \cdots$

then, $f''(a) =$

so, $c_2 =$

$f'''(x) = 2 \cdot 3c_3 + 2 \cdot 3 \cdot 4c_4(x-a) + \cdots + (n-2)(n-1)n \cdot c_n(x-a)^{n-3} + \cdots$

then, $f'''(a) =$

so, $c_3 =$

Do you see a pattern?
Predict $f^{(n)}(a) =$

so, $c_n =$

Therefore, the Power Series:

$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots + c_n(x-a)^n + \cdots$

can now be written as the

Taylor Series

$f(x) = \qquad + \qquad (x-a) + \qquad (x-a)^2 + \qquad (x-a)^3 + \cdots + \qquad (x-a)^n + \cdots$

$$= \sum_{k=}^{\infty} \qquad (x-a)^k$$

where f is generated (centered) @ $x = a$

and the partial sum, called the

Taylor Polynomial of order *n* for $f(x)$

$P_n(x) = \quad + \quad (x-a) + \quad (x-a)^2 + \quad (x-a)^3 + \cdots + \quad (x-a)^n$

$= \sum_{k=}^{n} \quad (x-a)^k$

where *f* is generated (centered) @ *x* = *a*

For a more specific case, we have:

Maclaurin Series

$f(x) = \quad + \quad x + \quad x^2 + \quad x^3 + \cdots + \quad x^n + \cdots$

$= \sum_{k=}^{\infty} \quad x^k$

where *f* is generated (centered) @ *x* = 0

Examples:

Find the Maclaurin Series of $f(x) = e^x$

Find the Taylor Series for $f(x) = e^x$ at *x* = 2.

Construct a fifth order Taylor Polynomial for $\sin x$ at $x = 0$.

Find a Taylor Polynomial of order 3 generated by $f(x) = \frac{1}{x}$ at $x = 1$.