DATE: ______________

9.2 Taylor/Maclaurin Series

Common Maclaurin Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$= 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots$$

converges $\forall$ real x

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$$

converges $\forall$ real x

$\cos x = \frac{d}{dx}(\sin x)$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$$

converges $\forall$ real x

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$= 1 + x + x^2 + \cdots + x^n + \cdots$$

converges when $|x|<1$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-x)^n \text{ or } \sum_{n=0}^{\infty} (-1)^n x^n$$

$$= 1 - x + x^2 - x^3 + \cdots + (-x)^n + \cdots$$

converges when $|x|<1$

$$\ln(1+x) = \cancel{\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

doesn't get 1st term

$$= x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{(-1)^{n+1} x^n}{n}$$

$\ln(1+x) = \int \frac{1}{1+x}\,dx$

$= x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots$

converges on $(-1, 1]$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$= x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots$$

$\tan^{-1} x = \int \frac{1}{1+x^2}\,dx$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-x^2)^n$

$= \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$= 1 - x^2 + x^4 + \cdots$

converges on $[-1, 1]$

Examples:

Write the first three non-zero terms and the general term of the Maclaurin series generated by the function.

1) $f(x) = \cos x^2$ $= \cos(x^2)$

$$= 1 - \frac{(x^2)^2}{2!} + \frac{(x^2)^4}{4!} + \cdots + \frac{(-1)^n (x^2)^{2n}}{(2n)!} + \cdots$$

$$= 1 - \frac{x^4}{2!} + \frac{x^8}{4!} + \cdots + \frac{(-1)^n x^{4n}}{(2n)!} + \cdots$$

2) $f(x) = x^2 e^x$ $= x^2\left(1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots\right)$

$$= x^2 + x^3 + \frac{x^4}{2!} + \cdots + \frac{x^{2+n}}{n!} + \cdots$$

3) $f(x) = \ln(1 + 3x)$ $= \ln(1 + (3x))$

$$= 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3} + \cdots + \frac{(-1)^{n+1}(3x)^n}{n} + \cdots$$

$$= 3x - \frac{9x^2}{2} + 9x^3 + \cdots + \frac{(-1)^{n+1}(3x)^n}{n} + \cdots$$

4) $f(x) = \frac{x}{1-2x}$ $= x\left(\frac{1}{1-(2x)}\right)$

$$= x\left(1 + 2x + (2x)^2 + \cdots + (2x)^n + \cdots\right)$$

$$= x + 2x^2 + 4x^3 + \cdots + 2^n x^{n+1} + \cdots$$

5) $f(x) = x \sin x$ $= x\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{(-1)\, x^{2n+1}}{(2n+1)!} + \cdots\right)$

$$= x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} + \cdots + \frac{(-1)\, x^{2n+2}}{(2n+1)!} + \cdots$$