DATE: ___________

AP Practice Power Series/Taylor Series

1. What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x = 0$ for $\sin x$?

(A) $1 - \frac{1}{2} + \frac{1}{24}$

(B) $1 - \frac{1}{2} + \frac{1}{4}$

(C) $1 - \frac{1}{3} + \frac{1}{5}$

(D) $1 - \frac{1}{4} + \frac{1}{8}$

(E) $1 - \frac{1}{6} + \frac{1}{120}$

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2. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0

(B) a_1

(C) $\sum_{n=0}^{\infty} a_n$

(D) $\sum_{n=1}^{\infty} n a_n$

(E) $\sum_{n=1}^{\infty} n {a_n}^{n-1}$

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3. The graph of the function represented by the Maclaurin series $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773

(B) 0.865

(C) 0.929

(D) 1.000

(E) 1.857

4. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin(x^2)$ is

(A) $-\frac{1}{6}$

(B) 0

(C) $\frac{1}{120}$

(D) $\frac{1}{6}$

(E) 1

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5. If $f(x) = \sum_{k=1}^{\infty}(\sin^2 x)^k$, then $f(1)$ is

(A) 0.369

(B) 0.585

(C) 2.400

(D) 2.426

(E) 3.426

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6. The interval of convergence of $\sum_{n=0}^{\infty}\frac{(x-1)^n}{3^n}$ is

(A) $-3 < x \leq 3$

(B) $-3 \leq x \leq 3$

(C) $-2 < x < 4$

(D) $-2 \leq x \leq 4$

(E) $0 \leq x \leq 2$

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AP Practice Power Series/Taylor Series

Let f be a function that has derivatives of all orders for all real numbers.
Assume $f(0) = 5$, $f'(0) = -3$, $f''(0) = 1$, and $f'''(0) = 4$.

(a) Write the third-degree Taylor polynomial for f about $x = 0$ and use it to approximate $f(0.2)$.

(b) Write the fourth-degree Taylor polynomial for g, where $g(x) = f(x^2)$, about $x = 0$.

(c) Write the third-degree Taylor polynomial for h, where $h(x) = \int_0^x f(t)dt$, about $x = 0$.

(d) Let h be defined as in part (c). Given that $f(1) = 3$, either find the exact value of $h(1)$ or explain why it cannot be determined.