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AP Practice Power Series/Taylor Series

1. What is the approximation of the value of $\sin 1$ obtained by using the fifth-degree Taylor polynomial about $x = 0$ for $\sin x$?

(A) $1 - \frac{1}{2} + \frac{1}{24}$

(B) $1 - \frac{1}{2} + \frac{1}{4}$

(C) $1 - \frac{1}{3} + \frac{1}{5}$

(D) $1 - \frac{1}{4} + \frac{1}{8}$

(E) $1 - \frac{1}{6} + \frac{1}{120}$

$P_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$P_5(1) = 1 - \frac{1}{3!} + \frac{1}{5!}$

$= 1 - \frac{1}{6} + \frac{1}{120}$

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2. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0

(B) a_1

(C) $\sum_{n=0}^{\infty} a_n$

(D) $\sum_{n=1}^{\infty} n a_n$

(E) $\sum_{n=1}^{\infty} n {a_n}^{n-1}$

$f(x) = \sum_{n=0}^{\infty} a_n x^n = 1 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots$

$f'(x) = a_1 + 2a_2 x + \cdots + n a_n x^{n-1} + \cdots$

$f'(1) = a_1 + 2a_2 + \cdots + n a_n (1)^{n-1} + \cdots$

$= a_1 + 2a_2 + \cdots + n a_n$

$= \sum_{n=1}^{\infty} n a_n$

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3. The graph of the function represented by the Maclaurin series $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773

(B) 0.865

(C) 0.929

(D) 1.000

(E) 1.857

$f(x) = 1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \cdots + \frac{(-x)^n}{n!} + \cdots$

$f(x) = e^{-x}$

$y = x^3$, $f(x) = e^{-x}$

intersect @ $x = .773$

4. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin(x^2)$ is

$\boxed{\textbf{(A)}}$ $-\frac{1}{6}$

(B) 0

(C) $\frac{1}{120}$

(D) $\frac{1}{6}$

(E) 1

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots +$

$\sin(x^2) = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} + \dots$

$= x^2 - \frac{x^6}{6} + \frac{x^{10}}{120} + \dots$

coefficient of $x^6 \rightarrow -\frac{1}{6}$

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5. If $f(x) = \sum_{k=1}^{\infty} (\sin^2 x)^k$, then $f(1)$ is

(A) 0.369

(B) 0.585

(C) 2.400

$\boxed{\textbf{(D)}}$ 2.426

(E) 3.426

$f(x) = \sum_{k=1}^{\infty} (\sin x)^{2k}$

$= (\sin x)^2 + (\sin x)^4 + \dots (\sin x)^{2k} + \dots$

$\frac{a}{1-r} = \frac{(\sin x)^2}{1-(\sin x)^2} = \frac{\sin^2 x}{1-\sin^2 x} = \frac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$

$f(1) = (\tan 1)^2 = 2.426$

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6. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

(A) $-3 < x \leq 3$

(B) $-3 \leq x \leq 3$

$\boxed{\textbf{(C)}}$ $-2 < x < 4$

(D) $-2 \leq x \leq 4$

(E) $0 \leq x \leq 2$

$\sum_{n=0}^{\infty} \left(\frac{x-1}{3}\right)^n$

$r = \frac{x-1}{3}$, so $\left|\frac{x-1}{3}\right| < 1$

$-1 < \frac{x-1}{3} < 1$

$-3 < x-1 < 3$

$-2 < x < 4$

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AP Practice Power Series/Taylor Series

Let f be a function that has derivatives of all orders for all real numbers.
Assume $f(0) = 5$, $f'(0) = -3$, $f''(0) = 1$, and $f'''(0) = 4$.

(a) Write the third-degree Taylor polynomial for f about $x = 0$ and use it to approximate $f(0.2)$.

(b) Write the fourth-degree Taylor polynomial for g, where $g(x) = f(x^2)$, about $x = 0$.

(c) Write the third-degree Taylor polynomial for h, where $h(x) = \int_0^x f(t)dt$, about $x = 0$.

(d) Let h be defined as in part (c). Given that $f(1) = 3$, either find the exact value of $h(1)$ or explain why it cannot be determined.

a) $P_3(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!}$

$f(x) = 5 - 3x + \frac{x^2}{2} + \frac{2x^3}{3}$

$f(0.2) = 5 - 3(.2) + \frac{(.2)^2}{2} + \frac{2(.2)^3}{3}$

$f(0.2) = 4.425$

b) $g(x) = f(x^2)$

$= 5 - 3(x^2) + \frac{(x^2)^2}{2}$

$g(x) = 5 - 3x^2 + \frac{x^4}{2}$

c) $h(x) = \int_0^x f(t)\,dt$

$= \int_0^x (5 - 3t + \frac{t^2}{2})\,dt$

$= (5t - \frac{3}{2}t^2 + \frac{1}{6}t^3)\Big|_0^x$

$= 5x - \frac{3}{2}x^2 + \frac{1}{6}x^3$

d) $h(1) = \int_0^1 f(t)\,dt$

Cannot find exact value of h(1) b/c only know f(1) and f(0),
so can't find exact function f(t) to antiderive
(could only approximate h(1) using Taylor polynomial)