HW5 – AP Taylor Series Practice

1. The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

a) Write a third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

b) Write the fourth-degree Taylor polynomial, $P(x)$. For $g(x) = f(x^2 + 2)$ about $x = 0$. Use P to explain why g must have a relative minimum at $x = 0$.

2. The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The *n*th derivative of f at $x = 5$ is given by $f^{(n)}(5) = \frac{(-1)^n n!}{2^n(n+2)}$ and $f(5) = \frac{1}{2}$. Write the third-degree Taylor polynomial for f about $x = 5$.

3. The Maclaurin series for the function f is given by:

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

a) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.

b) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$

4. Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

a) Find $P(x)$.

b) Find the coefficient of x^{22} in the Taylor series for f about $x = 0$.

c) Let G be the function given by $G(x) = \int_0^x f(t)\,dt$. Write the third-degree Taylor polynomial for G about $x = 0$.