DATE: ___________

9.3 Taylor's Theorem

Recall the Taylor Series, centered at x = a:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Recall Taylor's Polynomial approximation of $f(x)$:

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

then,

$f(x) - P_n(x) =$

$\leq$

<u>Error/Remainder</u>

$R_n(x) =$

LaGrange Error Bound

$|R_n(x)| = |f(x) - P(x)|$

$\leq$

$\leq$

Lagrange Error
Bound

Example 1:
Estimate the error of $\sin(0.2)$ from the Taylor Polynomial of order 4.

Example 2:
Find the error bound for the 5[th] degree polynomial approximation of *e*.

Example 3:
The approximation $\ln(1+x) \approx x - \frac{x^2}{2}$ is used when *x* is small. Use the Remainder Estimation Theorem to get a bound for the maximum error when $|x| \leq 0.1$.

Example 4:
What is the smallest order of Taylor polynomial centered at $x = 1$ which will approximate e^{x-1} on the domain $-1 \leq x \leq 3$ with LaGrange error bound less than 1?