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Lagrange Error Bound Practice

1. The function f has derivatives of all orders for all real numbers *x*. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

a) Write the third-degree Taylor polynomial for f about *x* = 2 and use it to approximate $f(1.5)$.

b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \leq 3$ for all *x* in the closed interval [1.5,2]. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \neq -5$.

2. Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for f about *x* = 0. Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| < \frac{1}{100}$

3. Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about *x* = 2 is given by $T(x) = 7 - 9(x-2)^2 - 3(x-2)^3$.

a) Use $T(x)$ to find an approximation for $f(0)$.

b) The fourth derivative of *f* satisfies the inequality $\left|f^{(4)}(x)\right| \leq 6$ for all *x* in the closed interval [0,2]. Use the Lagrange error bound on the approximation to $f(0)$