DATE: __________

Alternating Series Error Bound

Given

$$\sum(-1)^n a_n\,,$$

the remainder/error is found by looking at the first unused term

$$|R_n(x)| = |f(x) - P_n(x)|$$

remainder = function − polynomial approx

Because the series is alternating with individual terms that decrease in absolute value to zero, the remainder is less than or equal to the absolute value of the first unused term.

$$|f(x) - P_n(x)| \le |a_{n+1}|$$

actual value   estimated value   1st unused term

Example 1:

Find the error involved in calculating the sum of the first six terms of the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

$$\underbrace{\frac{(-1)^1}{1!}+\frac{(-1)^2}{2!}+\cdots+\frac{(-1)^6}{6!}}_{\text{1st six terms}} + \underbrace{\frac{(-1)^7}{7!}}_{\text{1st unused term}}$$

alternating series w/ individual terms decreasing in abs value to zero.

By alt. series error bound, error $\le \left|\frac{(-1)^7}{7!}\right| = \frac{1}{7!}$

Example 2:

Find the error involved in calculating the sum of the first six terms of the series:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

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$$\frac{(-1)^0}{0!}+\frac{(-1)^1}{1!}+\frac{(-1)^2}{2!}+\frac{(-1)^3}{3!}+\cdots+\frac{(-1)^7}{7!}+\cdots$$

$$\cancel{1-1}+\underbrace{\frac{1}{2}+\cdots+\frac{(-1)^7}{7!}}_{\text{1st six terms}}+\underbrace{\frac{(-1)^8}{8!}}_{\text{1st unused term}}$$

now, not there, so don't count as terms

alternating series w/ individual terms decreasing in abs. value to 0.

By alt series error bound, error $\le \left|\frac{(-1)^8}{8!}\right| = \frac{1}{8!}$

Example 3:

Estimate the error of sin(0.2) from the Taylor Polynomial of order 4.

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin(0.2) = \sum_{n=0}^{\infty} \frac{(-1)^n (0.2)^{2n+1}}{(2n+1)!}$$

$$\sin(0.2) = \underbrace{0.2 - \frac{(0.2)^3}{3!}}_{\text{order 4}} + \underbrace{\frac{(0.2)^5}{5!}}_{\text{1st unused term}}$$

alternating series w/ individual terms decreasing in abs value to 0.

By alt series error bound, error $\le \left|\frac{(0.2)^5}{5!}\right| = 2.667 \times 10^{-6}$