DATE: ________________

9.4 Radius of Convergence

Two More Convergence Tests

Recall comparison test from section 8.4, if $0 \le f(x) \le g(x)$

1) and $\int g(x)dx$ converges, then $\int f(x)dx$ also converges.
2) and $\int f(x)dx$ diverges, then $\int g(x)dx$ also diverges.

Direct Comparison Test

If $\sum a_n$ has no negative terms,

① $a_n \le c_n$ and $\sum c_n$ converges, then $\sum a_n$ ________________

② $d_n \le a_n$ and $\sum d_n$ diverges, then $\sum a_n$ ________________

Ratio Test

If $\sum a_n$ has no negative terms and

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$$

then,

① if $L < 1$, then the series ______________

② if $L > 1$, then the series ______________

③ if $L = 1$, then the test fails.

Determine if the series converges or diverges.

Example 1

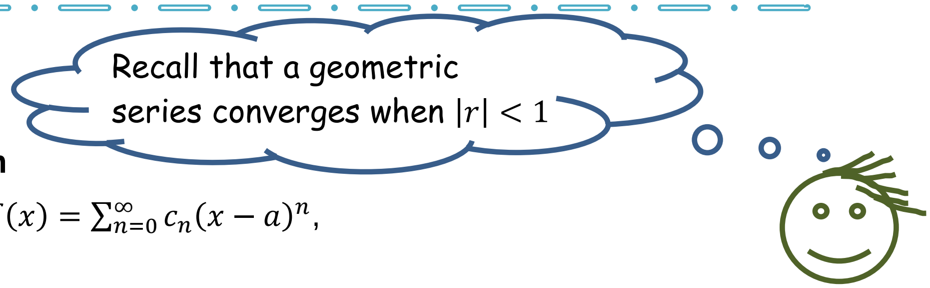
$$\sum_{n=1}^{\infty}\frac{1}{n^2+1}$$

Example 2

$$\sum_{n=0}^{\infty} n^2 e^{-n}$$

Example 3

$$\sum_{n=0}^{\infty} n!\, e^{-n}$$



Radius of Convergence Theorem

Convergence for a Power Series, $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$,

occurs in one of 3 ways:

① the series converges ∀x ____________

② ∃ a # R > 0 s.t. the series converges absolutely if _______________

and the series diverges absolutely if ________________

③ the series converges only at x = a ___________

Find the radius of convergence and the interval of convergence.

Example 1

$$\sum_{n=0}^{\infty} 2^n x^{n+2}$$

Example 2

$$\sum_{n=0}^{\infty} (x+5)^n$$

Example 3

$$\sum_{n=0}^{\infty} \frac{2^n}{n+1}$$

Example 4

$$\sum_{n=0}^{\infty} \frac{n}{2^n}(x-3)^n$$

Example 5

$$\sum_{n=1}^{\infty} n!\,(x-2)^n$$