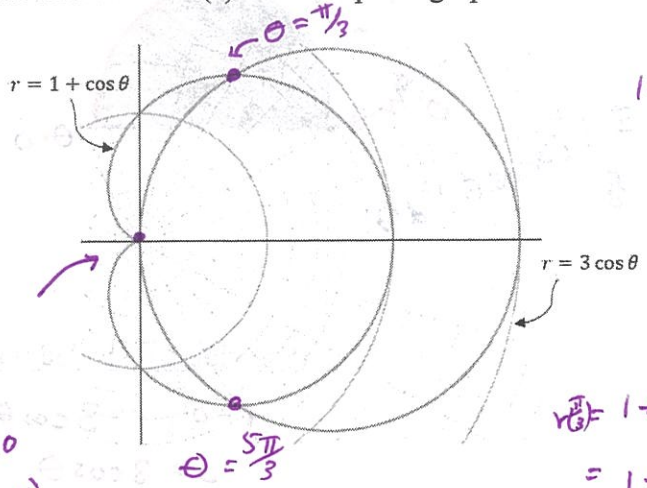
DATE: _____

Area Between Polar Curves

Find the intersections(s) for each polar graph.

1.



why not here?

b/c $r = 1+\cos\theta = 0$ when $\cos\theta = -1$, $\theta = \pi$

and $r = 3\cos\theta = 0$, $\cos\theta = 0$ when $\theta = \pi/2$ → not $r=0$ at the same θ.

$1 + \cos\theta = 3\cos\theta$

$1 = 2\cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$r(\frac{\pi}{3}) = 1 + \cos\frac{\pi}{3}$

$= 1 + \frac{1}{2}$

$= 3/2$

$(3/2, \pi/3)$

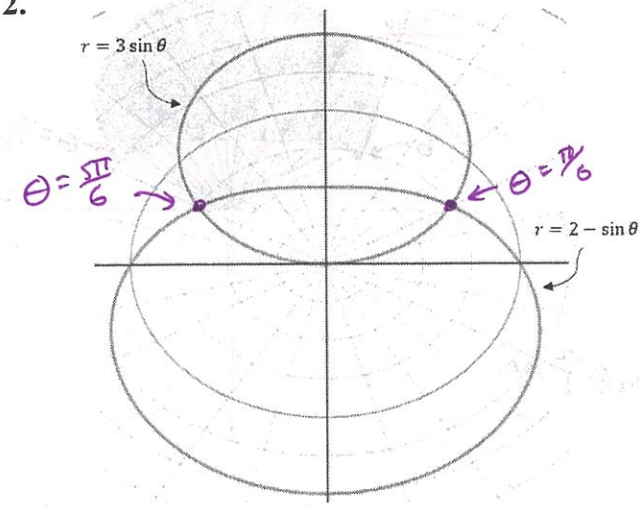
$r(\frac{5\pi}{3}) = 1 + \cos\frac{5\pi}{3}$

$= 1 + 1/2$

$= 3/2$

$(3/2, \frac{5\pi}{3})$

2.



$3\sin\theta = 2 - \sin\theta$

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = \pi/6, \frac{5\pi}{6}$

$r(\frac{\pi}{6}) = 3\sin\pi/6$

$= 3(\frac{1}{2})$

$= \frac{3}{2}$

$r(\frac{5\pi}{6}) = 3\sin\frac{5\pi}{6}$

$= 3/2$

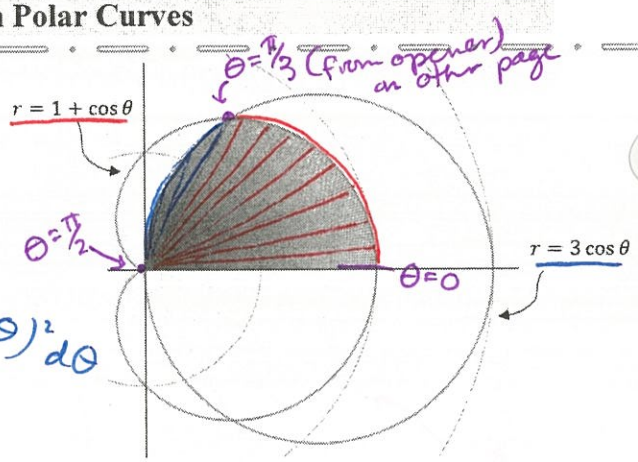
$(\frac{3}{2}, \frac{\pi}{6})$

$(\frac{3}{2}, \frac{5\pi}{6})$

Finding the Area Between Polar Curves

Example 1:

Find the area of the shaded region.



$$\text{Area} = \frac{1}{2}\int_0^{\pi/3}(1+\cos\theta)^2\,d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2}(3\cos\theta)^2\,d\theta$$

$$= 1.963$$

(See video of shaded region)

$r=0,\ r=3\cos\theta$

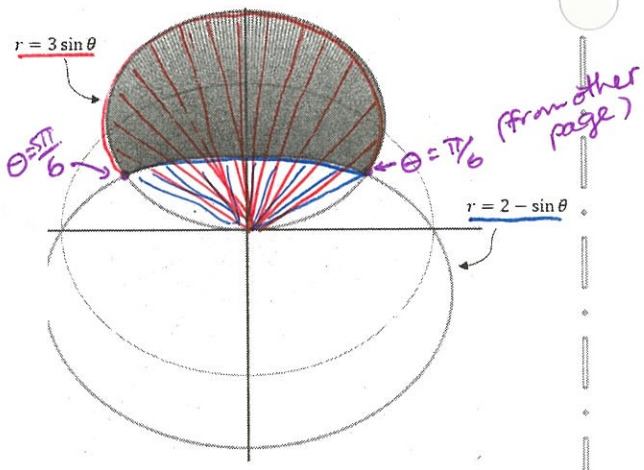
$0 = 3\cos\theta$

$\cos\theta = 0$

$\theta = \pi/2$

Example 2:

Find the area of the shaded region.



$$\text{Area} = \text{Red Area} - \text{Blue Area}$$

$$\text{Area} = \frac{1}{2}\int_{\pi/6}^{5\pi/6}(3\sin\theta)^2\,d\theta - \int_{\pi/6}^{5\pi/6}(2-\sin\theta)^2\,d\theta$$

$$= 5.196$$

(See shading in video1, video2)