DATE: _____

Radius of Convergence M/C Practice

1. Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{n^2+1}{n^5-n^2\sqrt{3}}$?

I. This series converges because $\lim_{n\to\infty} \frac{n^2+1}{n^5-n^2\sqrt{3}} = 0$.

II. This series converges by the Ratio Test.

(A) I only

(B) II only

(C) Both I and II

(D) Neither I nor II

I. *n^{th} term test*

$$\lim_{n\to\infty} \frac{n^2+1}{n^5-n^2\sqrt{3}} = 0$$

$\therefore$, n^{th} term test is inconclusive.

II. *Ratio Test*

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2+1}{(n+1)^5-(n+1)^2\sqrt{3}}}{\frac{n^2+1}{n^5-n^2\sqrt{3}}} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^2+1}{(n+1)^5-(n+1)^2\sqrt{3}} \cdot \frac{n^5-n^2\sqrt{3}}{n^2+1} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^2-1}{n^2+1} \cdot \frac{n^5-n^2\sqrt{3}}{(n+1)^5-(n+1)^2\sqrt{3}} \right|$$

$$= 1$$

$\therefore$, Ratio Test is inconclusive

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2. Which of the following series converge?

I. $\sum_{n=0}^{\infty} \frac{5n}{2n+1}$

II. $\sum_{n=1}^{\infty} \frac{e^n}{n}$

III. $\sum_{n=0}^{\infty} \frac{e^n+1}{e^n}$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) None

I. *n^{th} term test*

$$\lim_{n\to\infty} \frac{5n}{2n+1} = \frac{5}{2} \neq 0$$

$\therefore$, $\sum_{n=0}^{\infty} \frac{5n}{2n+1}$ diverges by the nth term test.

II. *Ratio Test*

$$\lim_{n\to\infty} \left| \frac{\frac{e^{n+1}}{n+1}}{\frac{e^n}{n}} \right| = \lim_{n\to\infty} \left| \frac{e^{n+1}}{n+1} \cdot \frac{n}{e^n} \right|$$

$$= \lim_{n\to\infty} \left| e \cdot \frac{n}{n+1} \right|$$

$$= e > 1$$

$\therefore$, $\sum_{n=1}^{\infty} \frac{e^n}{n}$ diverges by Ratio Test

III. *Direct Comparison Test*

$$\frac{e^n}{e^n} \leq \frac{e^n+1}{e^n}$$

$$0 \leq 1 \leq \frac{e^n+1}{e^n}$$

$\sum_{n=0}^{\infty} 1$ diverges by n^{th} term test

$\lim_{n\to\infty} 1 \neq 0$

$\therefore$ $\sum_{n=0}^{\infty} \frac{e^{n+1}}{e^n}$ also diverges by direct comparison test

OR *n^{th} term test*

$$\sum_{n=0}^{\infty} \frac{e^{n+1}}{e^n}$$

$$\lim_{n\to\infty} \frac{e^{n+1}}{e^n} = 1 \neq 0,$$

$\therefore$ $\sum_{n=0}^{\infty} \frac{e^{n+1}}{e^n}$ diverges by n^{th} term test

3. The radius of convergence for the series $\sum_{n=0}^{\infty} \frac{n^2(x-10)^n}{10^n}$ is

(A) 1

(B) $\frac{1}{10}$

(C) 10

(D) $\frac{n}{10}$

(E) ∞

* Ratio Test *

$$\lim_{n\to\infty}\left|\frac{\frac{(n+1)^2(x-10)^{n+1}}{10^{n+1}}}{\frac{n^2(x-10)^n}{10^n}}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^2(x-10)^{n+1}}{10^{n+1}}\cdot\frac{10^n}{n^2(x-10)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x-10}{10}\cdot\frac{(n+1)^2}{n^2}\right|$$

$$= \left|\frac{x-10}{10}\right|$$

Series converges when $\left|\frac{x-10}{10}\right| < 1$

$|x-10| < 10$ ← radius of convergence

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4. The radius of convergence for the series $\sum_{n=0}^{\infty} \frac{(x-3)^n}{n!}$ is

(A) 0

(B) $\frac{1}{n!}$

(C) 1

(D) 3

(E) ∞

* Ratio Test *

$$\lim_{n\to\infty}\left|\frac{\frac{(x-3)^{n+1}}{(n+1)!}}{\frac{(x-3)^n}{n!}}\right| = \lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{(n+1)!}\cdot\frac{n!}{(x-3)^n}\right|$$

$$= \lim_{n\to\infty}\left|(x-3)\cdot\frac{1}{n+1}\right|$$

$= 0$ for any x-value

$0 < 1$, radius of convergence is ∞ (series converges $\forall x$)

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5. The radius of convergence for the series $\sum_{n=0}^{\infty} \frac{(x-5)^n}{\sqrt{n}}$ is

(A) 0

(B) 1

(C) $\sqrt{5}$

(D) 5

(E) ∞

* Ratio Test *

$$\lim_{n\to\infty}\left|\frac{\frac{(x-5)^{n+1}}{\sqrt{n+1}}}{\frac{(x-5)^n}{\sqrt{n}}}\right| = \lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{(x-5)^n}\right|$$

$$= \lim_{n\to\infty}\left|(x-5)\cdot\frac{\sqrt{n}}{\sqrt{n+1}}\right|$$

$$= |x-5|$$

series converges when $|x-5| < 1$ ← radius of convergence