

### 9.4 Sequences & Series

Target 7D: Calculate the sums of finite and infinite series

Review of Prior Concepts

1. Find the 100<sup>th</sup> term in the following sequence of numbers.

a) 2, 5, 8, 11, ...  $a_n = 2 + (n-1)3$   
 $a_{100} = 3(100) - 1 = \boxed{299}$

b) 2, 4, 8, 16, ...

$a_n = 2(2)^{n-1}$   
 $a_{100} = 2(2)^{100-1} = \boxed{2^{100}}$

2. Find the sum of the 1<sup>st</sup> 100 positive integers.

$1+2+\dots+100 = \boxed{5050}$

### More Practice

Arithmetic and Geometric Sequences

<https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html>

<http://www.mathsisfun.com/algebra/sequences-sums-geometric.html>



### SAT Connection

#### Heart of Algebra

8. Interpret the variables and constants in expressions for linear functions within the context presented.

Example:

$$a = 18t + 15$$

Jane made an initial deposit to a savings account. Each week thereafter she deposited a fixed amount to the account. The equation above models the amount  $a$ , in dollars, that Jane has deposited after  $t$  weekly deposits. According to the model, how many dollars was Jane's initial deposit? (Disregard the \$ sign when gridding your answer.)

$a = 18t + 15$   
 $a = 15 + 18t$  arithmetic explicit rule  
 $\downarrow$   
 $a_1 = 15$  initial deposit

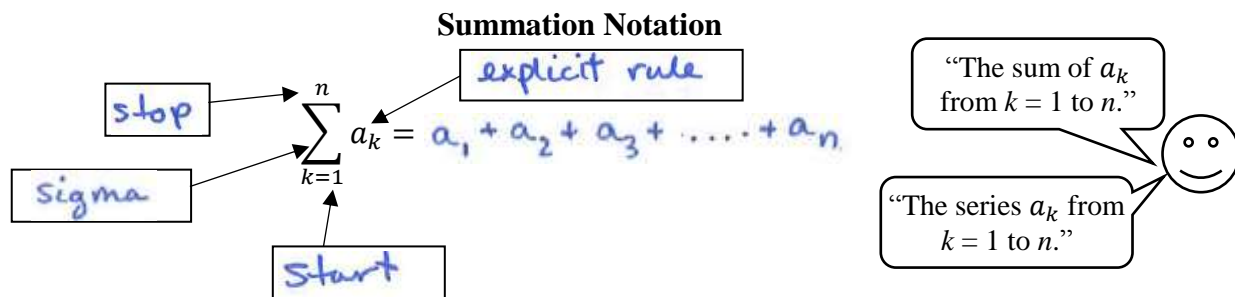
				1	5
/	○	○			
.	○	○	○	○	
0	○	○	○	○	
1	○	○	●	○	
2	○	○	○	○	
3	○	○	○	○	
4	○	○	○	○	
5	○	○	○	●	
6	○	○	○	○	
7	○	○	○	○	
8	○	○	○	○	
9	○	○	○	○	

**NOTE:** You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Solution

Summation/Series

Summation (or Series) - sum up the terms of a sequence



Example 1: Find the value of:

$$\sum_{k=2}^5 3k = 3(2) + 3(3) + 3(4) + 3(5) = 42$$

MENU, CALCULUS, SUM ... 😊

Example 2: Write the summation  $2 + 5 + 8 + 11 + \dots + 29$  in sigma notation.

Start  $k=1$

$\sum_{k=1}^{10} (3n-1)$

$a_n \leftarrow$  how many terms?

$$a_n = a_1 + (n-1)d$$

$$29 = 2 + (n-1)3$$

$$29 = 2 + 3n - 3$$

$$29 = 3n - 1$$

$$30 = 3n$$

$$10 = n$$

Arithmetic explicit rule

$$a_n = a_1 + (n-1)d$$

$$a_n = 2 + (n-1)3$$

$$= 2 + 3n - 3$$

$$a_n = 3n - 1$$

Example 3: Write the summation  $5 - 15 + 45 - 135 + \dots$  in sigma notation.

Start  $k=1$

$$\sum_{k=1}^{\infty} 5(-3)^{n-1}$$

or

$$\sum_{k=1}^{\infty} \frac{5}{3}(-3)^n$$

$\leftarrow$  how many terms?

$\infty$

Geometric explicit rule

$$a_n = a_1 r^{n-1}$$

$$a_n = 5(-3)^{n-1}$$

or

$$a_n = 5(-3)^n (-3)^{-1}$$

$$= \frac{5(-3)^n}{-3}$$

$$a_n = -\frac{5}{3}(-3)^n$$

## Sum of the Terms in an Arithmetic Sequence

Proof

Start with the sum of an arithmetic sequence

$$\sum_{k=1}^n a_k = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-1)d)$$

Write the terms backwards, starting with  $a_n$ .

$$\sum_{k=1}^n a_k = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n-1)d)$$

Add the two equations together,

$$2 \sum_{k=1}^n a_k = a_1 + a_n + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n)$$

Simplify,

$$2 \sum_{k=1}^n a_k = n(a_1 + a_n)$$

Solve for sigma,

$$\sum_{k=1}^n a_k = \frac{n(a_1 + a_n)}{2}$$

 $\therefore$ , Formula for Sum of the Terms in an Arithmetic Sequence is:

$$\sum_{k=1}^n a_k = \frac{n}{2}(a_1 + a_n) \quad \text{OR} \quad \sum_{k=1}^n a_k = \frac{n}{2}(2a_1 + (n-1)d)$$

$$\frac{n}{2}(a_1 + a_1 + (n-1)d)$$

Example 1:

Find the sum of:  $2 + 5 + 8 + 11 + \cdots + 29$ 

$$\sum_{k=1}^{10} a_k = \frac{10}{2}(2 + 29) = 155$$

arithmetic w/  $d=3$ 

$$a_n = a_1 + (n-1)d$$

$$29 = 2 + (n-1)3$$

$$29 = 2 + 3n - 3$$

$$29 = 3n - 1$$

$$30 = 3n$$

$$10 = n$$

Example 2:

Find the sum of the sequence:  $-3, 1, 5, 9, \dots, 133$ 

$$\sum_{k=1}^{35} a_k = \frac{35}{2}(-3 + 133) = 2275$$

arithmetic w/  $d=4$ 

$$a_n = a_1 + (n-1)d$$

$$133 = -3 + (n-1)4$$

$$133 = -3 + 4n - 4$$

$$133 = 4n - 7$$

$$140 = 4n$$

$$35 = n$$

**More Practice****Arithmetic Series**

<https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html>

<http://www.purplemath.com/modules/series4.htm>

[https://www.khanacademy.org/math/algebra2/sequences-and-series/copy-of-seq-and-series/e/arithmetric\\_series](https://www.khanacademy.org/math/algebra2/sequences-and-series/copy-of-seq-and-series/e/arithmetric_series)

<https://youtu.be/cYw4MFWsB6c>

<https://youtu.be/xWHfQGBzgbC>

<https://youtu.be/UHkueFmPC6s>

**Homework Assignment**

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**SAT Connection****Solution**

The correct answer is 15. The amount,  $a$ , that Jane has deposited after  $t$  fixed weekly deposits is equal to the initial deposit plus the total amount of money Jane has deposited in the  $t$  fixed weekly deposits. This amount  $a$  is given to be  $a = 18t + 15$ . The amount she deposited in the  $t$  fixed weekly deposits is the amount of the weekly deposit times  $t$ ; hence, this amount must be given by the term  $18t$  in  $a = 18t + 15$  (and so Jane must have deposited 18 dollars each week after the initial deposit). Therefore, the amount of Jane's original deposit, in dollars, is  $a - 18t = 15$ .