9.4 Sequences & Series

Target 7D: Calculate the sums of finite and infinite series

Review of Prior Concepts

1. Find the value of (without calculator):

$$\sum_{k=5}^{9}(11-3k)$$

2. Find the value of (with calculator):

$$\sum_{k=0}^{12}\left(\frac{1}{2}\right)^{k}$$

3. Find the sum of: $2+1+\frac{1}{2}+\frac{1}{4}+\cdots$

4. Find the sum of: $1+2+4+8+16+\cdots$

More Practice

Arithmetic & Geometric Sequences and Series

https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html
http://www.mathsisfun.com/algebra/sequences-sums-geometric.html
http://www.purplemath.com/modules/series4.htm

SAT Connection

Passport to Advanced Math

10. Interpret parts of nonlinear expressions in terms of their context.

Example: Of the following four types of savings account plans, which option would yield exponential growth of the money in the account?

A) Each successive year, 2% of the initial savings is added to the value of the account.

B) Each successive year, 1.5% of the initial savings and $100 is added to the value of the account.

C) Each successive year, 1% of the current value is added to the value of the account.

D) Each successive year, $100 is added to the value of the account.

Solution

Finite Geometric Series

$$\sum_{k=1}^{n} a_1 r^{k-1} = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^n$$

Formula for Sum of the Terms in a Finite Geometric Sequence is:

$$\sum_{k=1}^{n} a_1 r^{k-1} = \frac{a_1(1 - r^n)}{1 - r}$$

Example 1: Find the sum of the sequence: $2, 6, 18, \ldots, 2 \cdot 3^8$

Example 2: Find the sum: $4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \frac{4}{81}$

Infinite Geometric Series

$$\sum_{k=1}^{\infty} a_1 r^{k-1} = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^n + \cdots$$

Formula for Sum of the Terms of an Infinite Geometric Sequence is:

$$\sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1 - r} \qquad \text{when } |r| < 1$$

Example 1: Find the sum of the sequence: $2, 1, \frac{1}{2}, \frac{1}{4} \ldots$

Example 2: Find the sum: $\frac{1}{48} + \frac{1}{16} + \frac{3}{16} + \frac{9}{16} + \cdots$

Now, you try…

1. Find the sum of the sequence: $3, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}, \ldots$

2. Find:

$$\sum_{k=1}^{\infty}(-2)^k$$

3. Find the sum: $\frac{1}{16} - \frac{1}{48} + \frac{1}{144} - \cdots$

More Practice

Geometric Series

http://www.purplemath.com/modules/series5.htm
https://www.khanacademy.org/math/algebra2/sequences-and-series/copy-of-geometric-sequence-series/v/geometric-series-introduction
http://www.mathsisfun.com/algebra/sequences-sums-geometric.html
https://youtu.be/yYxzq_O18Mg
https://youtu.be/-JH5XSvJFTA
https://youtu.be/DO1bIuqFIDQ
https://youtu.be/haK3oC0L_a8

Homework Assignment

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SAT Connection
Solution

Choice C is correct. Let I be the initial savings. If each successive year, 1% of the current value is added to the value of the account, then after 1 year, the amount in the account will be $I + 0.01I = I(1 + 0.01)$; after 2 years, the amount in the account will be $I(1+0.01) + 0.01I(1+0.01) = (1+0.01)I(1+0.01) = I(1+0.01)^2$; and after t years, the amount in the account will be $I(1 + 0.01)^t$. This is exponential growth of the money in the account.

Choice A is incorrect. If each successive year, 2% of the initial savings, I, is added to the value of the account, then after t years, the amount in the account will be $I + 0.02It$, which is linear growth. Choice B is incorrect. If each successive year, 1.5% of the initial savings, I, and \$100 is added to the value of the the account, then after t years the amount in the account will be $I + (0.015I + 100)t$, which is linear growth. Choice D is incorrect. If each successive year, \$100 is added to the value of the account, then after t years the amount in the account will be $I + 100t$, which is linear growth.