9.4 Sequences & Series

Target 7D: Calculate the sums of finite and infinite series

Review of Prior Concepts

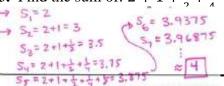
1. Find the value of (without calculator):

$$\sum_{k=5}^{9} (11-3k) = \{1-3(5)+11-3(6)+11-3(7) + (1-3(8)+11-3(7)) + (1-3(8)+11-3(7)) = -4-7-10-13-14 = -50\}$$

2. Find the value of (with calculator):

$$\sum_{k=0}^{12} \left(\frac{1}{2}\right)^k = \boxed{\frac{8191}{4096}}$$

3. Find the sum of: $2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$



4. Find the sum of: $1 + 2 + 4 + 8 + 16 + \cdots$

More Practice

Arithmetic & Geometric Sequences and Series

https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html http://www.mathsisfun.com/algebra/sequences-sums-geometric.html http://www.purplemath.com/modules/series4.htm



SAT Connection

Passport to Advanced Math

10. Interpret parts of nonlinear expressions in terms of their context. .

Example:

Of the following four types of savings account plans, which option would yield exponential growth of the money in the account?

- Each successive year, 2% of the initial savings is added to the value of the account.
- B) Each successive year, 1.5% of the initial savings and \$100 is added to the value of the account.
- Each successive year, 1% of the current value is added to the value of the account.
- Each successive year, \$100 is added to the value of the account.

(A)
$$a_n = 0.2(a_1) + a_{n-1}$$

(c)
$$a_n = 0.1(a_{n-1}) + a_{n-1}$$

$$= a_{n-1}(0.1+1)$$

$$= 0.1(a_{n-1}) + a_{n-1}$$

$$= a_{n-1}(0.1+1)$$

$$= 0.1(a_{n-1}) + a_{n-1}$$

$$= a_{n-1}(0.1+1)$$

Solution

Finite Geometric Series

$$\sum_{k=1}^{n} a_1 r^{k-1} = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^n$$

Formula for Sum of the Terms in a Finite Geometric Sequence is:

$$\left[\sum_{k=1}^{n} a_1 r^{k-1} = \frac{a_1 (1 - r^n)}{1 - r} \right]$$

Example 1: Find the sum of the sequence:
$$\frac{2}{2}$$
, 6, 18, ..., $\frac{2}{2} \cdot 3^8$

$$\frac{2}{2} \cdot 3^{n-1} = \frac{2(1-3^9)}{1-3}$$

$$\frac{2(1-3^9)}{1-2}$$

$$\frac{2(1$$

Example 2: Find the sum: $4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \frac{4}{81}$

$$\sum_{k=1}^{5} 4(-\frac{1}{3})^{k-1} = \frac{4(1-(-\frac{1}{3})^{\frac{1}{3}})}{1-\frac{1}{3}} = \frac{4(1-(-\frac{1}{2}\sqrt{3}))}{1+\frac{1}{3}}$$

$$= \frac{4(1-(-\frac{1}{2}\sqrt{3}))}{1+\frac{1}{3}}$$

$$= \frac{4(1+\frac{1}{2}\sqrt{3})}{4/3} = \boxed{244} \text{ or } \boxed{3.01235}$$

Infinite Geometric Series

$$\sum_{k=1}^{\infty} a_1 r^{k-1} = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^n + \dots$$

Formula for Sum of the Terms of an Infinite Geometric Sequence is:

$$\left[\sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1-r} \quad \text{when } |r| < 1\right]$$

Example 1: Find the sum of the sequence:
$$2+1+\frac{1}{2}+\frac{1}{4}+\cdots$$

$$\sum_{k=1}^{\infty} z\left(\frac{1}{2}\right)^{k-1} = \frac{2}{1-\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = 2 \cdot \frac{2}{1} = \boxed{4}$$
The series "constarges to 4"

Unit 7 (Chapter 9): Discrete Mathematics

Pre-Calculus 2016-2017

Example 2: Find the sum: $\frac{1}{48} + \frac{1}{16} + \frac{3}{16} + \frac{9}{16} + \cdots$

"The series diverges"

geometrice
$$w/r=3$$
Is $|3| < 1$?
$$3 < 1 \times$$

$$3 < 1 \times$$

$$3 < 1 \times$$

$$3 < 1 \times$$

Now, you try...

1. Find the sum of the sequence: $3, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}, \dots$

$$\sum_{k=1}^{\infty} 3(\frac{1}{4})^{k-1} = \frac{3}{1-\frac{1}{4}}$$

$$= \frac{3}{3\frac{1}{4}}$$

$$= 3 \cdot \frac{4}{3} = \boxed{4}$$
The series converges to 4

2. Find:

$$\sum_{k=1}^{\infty} (-2)^k = (-2)^k + (-2)^l + (-2)^l + \cdots$$

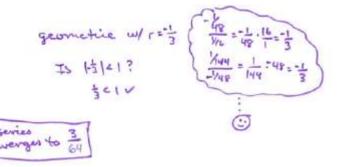
$$= 1 - 2 + 4 + \cdots$$
Since $|-2| \neq 1$, the series diverges.

3. Find the sum: $\frac{1}{16} - \frac{1}{48} + \frac{1}{144} - \cdots$

$$\sum_{k=1}^{\infty} \frac{1}{16} \left(\frac{1}{3}\right)^{k-1} = \frac{N_{16}}{1-V_{3}}$$

$$= \frac{N_{16}}{4V_{3}}$$

$$= \frac{1}{16} \cdot \frac{3}{4} = \frac{3}{64}$$
The series converges to $\frac{3}{64}$



More Practice

Geometric Series

http://www.purplemath.com/modules/series5.htm

https://www.khanacademy.org/math/algebra2/sequences-and-series/copy-of-geometric-sequence-

series/v/geometric-series-introduction

http://www.mathsisfun.com/algebra/sequences-sums-geometric.html

https://youtu.be/yYxzq_O18Mg

https://youtu.be/-JH5XSvJFTA

https://youtu.be/DO1bIuqFIDQ

https://youtu.be/haK3oC0L_a8

Homework Assignment

p.747 #49-59odd

SAT Connection

Solution

Choice C is correct. Let I be the initial savings. If each successive year, 1% of the current value is added to the value of the account, then after 1 year, the amount in the account will be I + 0.01I = I(1 + 0.01); after 2 years, the amount in the account will be $I(1 + 0.01) + 0.01I(1 + 0.01) = (1 + 0.01)I(1 + 0.01) = I(1 + 0.01)^2$; and after t years, the amount in the account will be $I(1 + 0.01)^t$. This is exponential growth of the money in the account.

Choice A is incorrect. If each successive year, 2% of the initial savings, I, is added to the value of the account, then after t years, the amount in the account will be I + 0.02It, which is linear growth. Choice B is incorrect. If each successive year, 1.5% of the initial savings, I, and \$100 is added to the value of the the account, then after t years the amount in the account will be I + (0.015I + 100)t, which is linear growth. Choice D is incorrect. If each successive year, \$100 is added to the value of the account, then after t years the amount in the account will be I + 100t, which is linear growth.