

9.5 Testing Convergence at Endpoints

Three More Convergence Tests

Integral Test

Suppose a_n is a sequence of positive terms and $a_n = f(n)$, where f is continuous, positive, decreasing function of $x \forall x \geq N$ (where N is positive integer),

① If $\int_N^{\infty} f(x) dx$ converges, then $\sum_{n=N}^{\infty} a_n$ _____

② If $\int_N^{\infty} f(x) dx$ diverges, then $\sum_{n=N}^{\infty} a_n$ _____

Determine if the series converges or diverges.

Example 1

$$\sum_{n=1}^{\infty} \frac{1}{1+10n}$$

Example 2

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

p -Series Test

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

① if $p > 1$, then the series _____

② if $0 < p < 1$, then the series _____

③ if $p = 1$, then the series _____

• Determine if the series converges or diverges.

Example 1

$$\sum_{n=1}^{\infty} n^{-2}$$

Example 2

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

Limit Comparison Test

Suppose $a_n > 0$ and $b_n > 0 \quad \forall n \geq N$ (where N is positive integer)

① If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c$ (where $0 < c < \infty$) and $\sum b_n$ converges, then $\sum a_n$ _____

or if $\sum b_n$ diverges, then $\sum a_n$ _____

② If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ _____

③ If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ _____

• Determine if the series converges or diverges.

Example 1

$$\sum_{n=1}^{\infty} \frac{3}{n+1}$$

Example 2

$$\sum_{n=1}^{\infty} \frac{1}{e^n + n}$$