DATE: ______________

AP Series Problems

1. A function f is defined by $f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$ for all x in the interval of convergence of the given power series. Find the interval of convergence for this power series. Show the work that leads to your answer.

2. The function f has a Taylor series about $x = 2$ that converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 2$ is given by $f^{(n)}(x) = \frac{(n+1)!}{3^n}$ for $n \geq 1$, and $f(2) = 1$.

a) Write the first four terms and the general term of the Taylor series for f about $x = 2$.

b) Find the radius of convergence for the Taylor series for f about $x = 2$. Show the work that leads to your answer.

c) Let g be a functions satisfying $g(2) = 3$ and $g'(x) = f(x)$ for all x. Write the first four terms and the general term of the Taylor series for g about $x = 2$.

d) Does the Taylor series for g as defined in part (c) converge at $x = -2$? Give a reason for your answer.

3. Let f be a function with derivatives of all orders and for which $f(2) = 7$. When n is odd, the nth derivative of f at $x = 2$ is 0. When n is even and $n \geq 2$, the nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \frac{(n-1)!}{3^n}$.

a) Write the sixth-degree Taylor polynomial for f about $x = 2$.

b) In the Taylor series for f about $x = 2$, what is the coefficient of $(x-2)^{2n}$ for $n \geq 1$?

c) Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.

4. The function f is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n n x^n}{n+1} + \cdots$$

for all real numbers x for which the series converges. The function g is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!} + \cdots$$

for all real numbers x for which the series converges.
Find the interval of convergence of the power series for f. Justify your answer.

5. The Maclaurin series for $\ln\left(\frac{1}{1-x}\right)$ is $\sum_{n=1}^{\infty} \frac{x^n}{n}$ with interval of convergence $-1 \leq x < 1$.

a) Find the Maclaurin series for $\ln\left(\frac{1}{1+3x}\right)$ and determine the interval of convergence.

b) Find the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

c) Give a value of p such that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges, but $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges. Give reasons why your value of p is correct.