

Convergence Practice

1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series test ($p=2 > 1$)

II. $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test ($p=1$)

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ alternating series test

- (A) I only
- (B) III only
- (C) I and II only
- (D) I and III only**
- (E) I, II, and III

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

① $\frac{1}{\sqrt{n}} > 0$

② $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$

③ $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges by alt. series test

2. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ converges?

- ~~(A)~~ $-1 \leq x < 1$
- ~~(B)~~ $-1 \leq x \leq 1$
- (C) $0 < x < 2$
- (D) $0 \leq x < 2$**
- (E) $0 \leq x \leq 2$

* Ratio Test *

$$\lim_{n \rightarrow \infty} \left| \frac{(x-1)^{n+1}}{n+1} \cdot \frac{n}{(x-1)^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(x-1)^{n+1}}{n+1} \cdot \frac{n}{(x-1)^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| (x-1) \frac{n}{n+1} \right|$$

$$= |x-1|$$

$$|x-1| < 1$$

$$-1 < x-1 < 1$$

$$0 < x < 2$$

$0 \leq x < 2$

check endpoints:

$x=2, \sum_{n=1}^{\infty} \frac{(2-1)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test

so, $x=2$ not included

$x=0, \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges by alt. series test

so $x=0$ is included

- b/c ① $\frac{1}{n} > 0$
- ② $\frac{1}{n} > \frac{1}{n+1}$
- ③ $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$

3. Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$

II. $\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{3}{2}\right)^n$

III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

- (A) I only
- (B) II only
- (C) III only
- (D) I and III only**
- (E) I, II, and III

I. alt. series test

① $\frac{1}{2n+1} > 0$

② $\frac{1}{2n+1} > \frac{1}{2(n+1)+1}$

$\frac{1}{2n+1} > \frac{1}{2n+3}$

③ $\lim_{n \rightarrow \infty} \frac{1}{2n+1} = 0$

$\therefore \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$

converges by alt.

4. The complete interval of convergence of the series

$$\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$$

is:

- (A) $0 < x < 2$
- (B) $0 \leq x \leq 2$
- (C) $-2 < x \leq 0$
- (D) $-2 \leq x < 0$
- (E) $-2 \leq x \leq 0$

5. What are all values of x for which the series

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$$

converges?

- (A) $-3 < x < -1$
- (B) $-3 \leq x < -1$
- (C) $-3 \leq x \leq -1$
- (D) $-1 \leq x < 1$
- (E) $-1 \leq x \leq 1$