DATE: ___________

## Convergence Practice

**1.** Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series test $(p = 2 > 1)$

II. $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test $(p = 1)$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ alternating series test

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

① $\frac{1}{\sqrt{n}} > 0$

② $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$

③ $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

$\therefore$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges by alt. series test

**(A)** I only

**(B)** III only

**(C)** I and II only

$\boxed{\textbf{(D)}}$ I and III only

**(E)** I, II, and III

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**2.** What are all values of $x$ for which the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ converges?

~~**(A)**~~ $-1 \le x < 1$

~~**(B)**~~ $-1 \le x \le 1$

**(C)** $0 < x < 2$

$\boxed{\textbf{(D)}}$ $0 \le x < 2$

**(E)** $0 \le x \le 2$

*Ratio Test*

$\lim_{n\to\infty} \left| \frac{\frac{(x-1)^{n+1}}{n+1}}{\frac{(x-1)^n}{n}} \right|$

$= \lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{n+1} \cdot \frac{n}{(x-1)^n} \right|$

$= \lim_{n\to\infty} \left| (x-1) \frac{n}{n+1} \right|$

$= |x-1|$

converges when $|x-1| < 1$

$-1 < x-1 < 1$

$0 < x < 2$

$\boxed{0 \le x < 2}$

check endpts:

$x = 2$, $\sum_{n=1}^{\infty} \frac{(2-1)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test

so, $x = 2$ not included

$x = 0$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges by alt. series test

b/c ① $\frac{1}{n} > 0$ ② $\frac{1}{n} > \frac{1}{n+1}$ ③ $\lim_{n\to\infty} \frac{1}{n} = 0$

so $x = 0$ is included

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**3.** Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$

II. $\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{3}{2}\right)^n$

III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

I. alt. series test

① $\frac{1}{2n+1} > 0$

② $\frac{1}{2n+1} > \frac{1}{2(n+1)+1}$

$\frac{1}{2n+1} > \frac{1}{2n+3}$

③ $\lim_{n\to\infty} \frac{1}{2n+1} = 0$

$\therefore$, $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$ converges by alt. series test

$\boxed{\textbf{(A)}}$ I only

**(B)** II only

**(C)** III only

**(D)** I and III only

**(E)** I, II, and III

II. Ratio Test

$\lim_{n\to\infty} \left| \frac{1}{n+1} \cdot \left(\frac{3}{2}\right)^{n+1} \cdot \frac{n}{\left(\frac{3}{2}\right)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{3/2\,(n)}{n+1} \right|$

$= \frac{3}{2} > 1$, so $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{3}{2}\right)^n$ diverges by Ratio test

III. Integral test

$\frac{1}{x \ln x}$ is cont, pos, and dec

$\int_2^{\infty} \frac{1}{x \ln x}\,dx$

$= \lim_{a\to\infty} \int_2^a \frac{1}{x \ln x}\,dx$

$= \lim_{a\to\infty} \ln(\ln x) \Big|_2^a$

$= \lim_{a\to\infty} (\ln(\ln a) - \ln(\ln 2))$

$= \infty$ diverges

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \frac{1}{u}\,du$

$\ln u$

$\therefore$, $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges by integral test

**4.** The complete interval of convergence of the series

$$\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$$

is:

- ~~**(A)**~~ $0 < x < 2$
- ~~**(B)**~~ $0 \le x \le 2$
- **(C)** $-2 < x \le 0$
- **(D)** $-2 \le x < 0$
- $\boxed{\textbf{(E)}}$ $-2 \le x \le 0$

*Ratio Test*

$$\lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(x+1)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x+1)\, n^2}{(n+1)^2} \right|$$

$$= |x+1|$$

$|x+1| < 1$ converges by ratio test

$-1 < x+1 < 1$

$-2 < x < 0$

$$\boxed{-2 \le x \le 0}$$

check endpts:

$x=0$, $\sum_{n=1}^{\infty} \frac{(0+1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series test ($p = 2 > 1$)

so, $x=0$ is included

$x=-2$, $\sum_{n=1}^{\infty} \frac{(-2+1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges by alt. series test b/c

① $\frac{1}{n^2} > 0$

② $\frac{1}{n^2} > \frac{1}{(n+1)^2}$

③ $\lim_{n\to\infty} \frac{1}{n^2} = 0$

so $x=-2$ is included

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**5.** What are all values of $x$ for which the series

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$$

converges?

- **(A)** $-3 < x < -1$
- $\boxed{\textbf{(B)}}$ $-3 \le x < -1$
- **(C)** $-3 \le x \le -1$
- ~~**(D)**~~ $-1 \le x < 1$
- ~~**(E)**~~ $-1 \le x \le 1$

*Ratio Test*

$$\lim_{n\to\infty} \left| \frac{(x+2)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(x+2)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x+2)\sqrt{n}}{\sqrt{n+1}} \right|$$

$$= |x+2|$$

$|x+2| < 1$ converges by ratio test

$-1 < x+2 < 1$

$-3 < x < -1$

$$\boxed{-3 \le x < -1}$$

check endpts:

$x=-1$, $\sum_{n=1}^{\infty} \frac{(-1+2)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by p-series test ($p = \frac{1}{2} < 1$)

so $x=-1$ not included

$x=-3$, $\sum_{n=1}^{\infty} \frac{(-3+2)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges by alt series test b/c

① $\frac{1}{\sqrt{n}} > 0$

② $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$

③ $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

so $x=-3$ is included