

Convergence Practice

1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

II. $\sum_{n=1}^{\infty} \frac{1}{n}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

- (A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III
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2. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ converges?

- (A) $-1 \leq x < 1$
(B) $-1 \leq x \leq 1$
(C) $0 < x < 2$
(D) $0 \leq x < 2$
(E) $0 \leq x \leq 2$
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3. Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$

II. $\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{3}{2}\right)^n$

III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

- (A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

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4. The Maclaurin series for $\ln\left(\frac{1}{1-x}\right)$ is $\sum_{n=1}^{\infty} \frac{x^n}{n}$ with interval of convergence $-1 \leq x < 1$.
- a) Find the Maclaurin series for $\ln\left(\frac{1}{1+3x}\right)$ and determine the interval of convergence.
- b) Find the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
- c) Give a value of p such that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges, but $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ diverges. Give reasons why your value of p is correct.
- d) Give a value of p such that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges, but $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges. Give reasons why your value of p is correct.