Convergence Practice

1. Which of the following series converge?
   
   I. \[ \sum_{n=1}^{\infty} \frac{1}{n^2} \quad \text{p-series, } p>1, \text{ converges} \]
   
   II. \[ \sum_{n=1}^{\infty} \frac{1}{n} \quad \text{p-series, } p=1, \text{ diverges} \]
   
   III. \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \quad \text{Alternating Series Test, } \frac{1}{\sqrt{n}} \text{ decreases and } u_n \to 0 \]

   (A) I only
   (B) III only
   (C) I and II only
   (D) I and III only
   (E) I, II, and III

2. What are all values of \( x \) for which the series \[ \sum_{n=1}^{\infty} \frac{(x-1)^n}{n} \] converges?

   (A) \(-1 \leq x < 1\)
   (B) \(-1 \leq x \leq 1\)
   (C) \(0 < x < 2\)
   (D) \(0 \leq x < 2\)
   (E) \(0 \leq x \leq 2\)

3. Which of the following series converge?
   
   I. \[ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1} \]
   
   II. \[ \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{3}{2} \right)^n \]
   
   III. \[ \sum_{n=2}^{\infty} \frac{1}{n \ln n} \]

   (A) I only
   (B) II only
   (C) III only
   (D) I and III only
   (E) I, II, and III
4. The Maclaurin series for $\ln \left( \frac{1}{1-x} \right)$ is $\sum_{n=1}^{\infty} \frac{x^n}{n}$ with interval of convergence $-1 \leq x < 1$.

a) Find the Maclaurin series for $\ln \left( \frac{1}{1+3x} \right)$ and determine the interval of convergence.

b) Find the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

c) Give a value of $p$ such that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges, but $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges. Give reasons why your value of $p$ is correct.

d) Give a value of $p$ such that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges, but $\sum_{n=1}^{\infty} \frac{1}{n^{3p}}$ converges. Give reasons why your value of $p$ is correct.