Summary of Convergence Tests

TEST	SERIES	Series CONVERGES if…	Series DIVERGES if …	COMMENTS
n[th] term Test	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	• should be 1st test used. • inconclusive if $\lim_{n\to\infty} a_n = 0$
Geometric Series Test	$\sum_{n=1}^{\infty} a_n r^{n-1}$ OR $\sum_{n=0}^{\infty} a_n r^n$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	• use when have "common ratio" • $\sum_{n=0}^{\infty} a_n r^n = \frac{a}{1-r}$
p-Series Test	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$0 < p \leq 1$	• called harmonic series when $p = 1$ • useful for comparison tests
Integral Test	$\sum_{n=1}^{\infty} a_n$ $a_n = f(x)$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	• f(x) must be continuous, positive, and decreasing
Direct Comparison Test	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 \leq b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	• to show convergence, find a larger series • to show divergence, find a smaller series
Limit Comparison Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L \geq 0$ (L is #) and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	• choose a b_n that "behaves" like a_n
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		• must prove terms are decreasing and limit equals zero • $\lvert \text{Error} \rvert \leq a_{n+1}$
Ratio Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	• inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$