AP M/c 2003

Non-Calculator

(1) $\frac{dy}{dx} = 2(x^3+1)(3x^2)$ chain rule

$= 6x^2(x^3+1)$

$\boxed{E}$

(2) $\int_0^1 e^{-4x}\,dx$

$u = -4x$

$du = -4\,dx$

$\frac{du}{-4} = dx$

$u(0) = -4(0) = 0$

$u(1) = -4(1) = -4$

$= \int_0^{-4} e^u \cdot \frac{du}{-4}$

$= \frac{1}{-4}\int_0^{-4} e^u\,du$

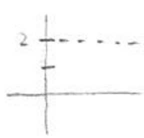
$= -\frac{1}{4}e^u \Big|_0^{-4}$

$= -\frac{1}{4}\left[e^{-4} - e^0\right]$

$= -\frac{1}{4}(e^{-4}-1) \Rightarrow -\frac{1}{4}e^{-4} + \frac{1}{4} \Rightarrow \frac{1}{4} - \frac{e^{-4}}{4}$

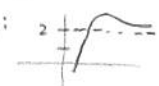
$\boxed{D}$

(3) horizontal asym. @ $y=2$, for $x \geq 0$



a) $f(0)=2$, false b/c don't know y-value @ $x=0$

b) $f(x) \neq 2$, for all $x \geq 0$ false, graph might look like:



c) $f(2)$ undefined, false don't know when $x=2$ what the y-value is.

d) $\lim_{x\to 2} f(x) = \infty$, false graph will follow asymptote not keep going up to ∞.

$\boxed{E}$ e) $\lim_{x\to\infty} f(x) = 2$, true as x goes to infinity, the graph goes to 2.

(4) $\frac{dy}{dx} = \frac{(3x+2)(2)-(2x+3)(3)}{(3x+2)^2}$ quotient rule.

$= \frac{6x+4-6x-9}{(3x+2)^2}$

$= \frac{-5}{(3x+2)^2}$ $\boxed{D}$

(5) $\int_0^{\pi/4} \sin x \, dx$

$= -\cos x \Big|_0^{\pi/4}$

$= -\cos \pi/4 - (-\cos 0)$

$= -\frac{\sqrt{2}}{2} + 1$ $\boxed{D}$

(6) $\lim_{x\to\infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} = \frac{1}{4}$

Same power, so take coefficients.

or

$\lim_{x\to\infty} \frac{x^3/x^3 - 2x^2/x^3 + 3x/x^3 - 4/x^3}{4x^3/x^3 - 3x^2/x^3 + 2x/x^3 - 1/x^3}$ divide by highest power.

$= \lim_{x\to\infty} \frac{1 - 2/x + 3/x^2 - 4/x^3}{4 - 3/x + 2/x^2 - 1/x^3}$ $\to 0$

$= \frac{1}{4}$ $\boxed{C}$

(7) Looking at graph of derivative → look where above & below x-axis to get f.

f': $[-2, 0]$ $+$, $[0, 2]$ $-$

f inc $(-2, 0)$, f dec. $(0, 2)$

(A) false - inc & dec.

(B) true $\boxed{B}$

(C) false - dec.

(D) false - max

(E) false b/c f' has y-values

(8) $\int x^2 \cos(x^3)\, dx$

$u = x^3$

$du = 3x^2\, dx$

$\frac{du}{3x^2} = dx$

$\int x^2 \cos u \cdot \frac{du}{3x^2}$

$= \frac{1}{3}\int \cos u \, du$

$= \frac{1}{3} \sin u + C$

$= \frac{1}{3} \sin(x^3) + C$ $\boxed{B}$

(9) $f(x) = \ln(x + 4 + e^{-3x})$

$f'(x) = \frac{1}{x + 4 + e^{-3x}} \cdot (1 + e^{-3x}(-3))$

$f'(0) = \frac{1}{0 + 4 + e^0} \cdot (1 + e^0 \cdot -3)$

$= \frac{1}{4+1}(1 - 3)$

$= \frac{-2}{5}$ $\boxed{A}$

(10) $f(x) < 0$ → all pts below x-axis

$f'(x) < 0$ → decreasing

$f''(x) < 0$ → concave down.

(A) no, inc & dec.

(B) yes $\boxed{B}$

(C) no, inc

(D) no, conc. up & down

(E) no, pts above x-axis

(11) $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\, dx$

$u = 2x + 1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$u(0) = 2(0) + 1 = 1$

$u(2) = 2(2) + 1 = 5$

$= \int_1^5 \sqrt{u} \cdot \frac{du}{2}$

$= \frac{1}{2} \int_1^5 \sqrt{u}\, du$ $\boxed{C}$

(12) rate of change of volume w/respect to time $\rightarrow \frac{dV}{dt}$

directly proportional $\rightarrow k \cdot \sqrt{V}$
to sq. root of volume

$$\frac{dV}{dt} = k\sqrt{V} \quad \boxed{E}$$

(13) f → cont ; no holes, jumps, or asymptotes.

f → diff'able ; no jumps, asymptotes, sharp turns, or vertical tangents

(A) a → cont, but sharp turn $\boxed{A}$

(B) b → not cont. (hole)

(c) c → cont, diff'able

(d) d → not cont (jump)

(e) e → cont, diff'able

(14) $\frac{dy}{dx} = x^2(\cos 2x)\cdot 2 + \sin 2x \cdot 2x$ product rule.

$= 2x^2 \cos 2x + 2x \sin 2x$

$= 2x(x\cos 2x + \sin 2x)$ $\boxed{E}$

(15)

$f'(x) = x^2 - \frac{2}{x}$

$0 = x^2 - \frac{2}{x}$, $x \neq 0$

$\frac{2}{x} = x^2$

$x^3 = 2$

$x = \sqrt[3]{2}$

f' + − +

-1 0 .1 $\sqrt[3]{2}$ 2

dec. $(0, \sqrt[3]{2}]$ $\boxed{D}$

(16) $f'(1) = \frac{f(1) - f(-2)}{1-(-2)}$ slope!

$= \frac{7-(-2)}{1-(-2)}$

$= \frac{9}{3}$

$= 3$ $\boxed{C}$

(17) $f(x) = 2xe^x$ concavity $\rightarrow f''(x)$!

$f'(x) = 2xe^x + e^x \cdot 2$ product rule

$= 2e^x(x+1)$

$F''(x) = 2e^x(1) + (x+1)(2e^x)$

$= 2e^x(1 + x + 1)$

$= 2e^x(x+2)$

$2e^x(x+2) = 0$

$\cancel{2e^x = 0}$ $x + 2 = 0$

$x = -2$

f'' − +

-3 -2 0

conc. down $(-\infty, -2)$

or $x < -2$ $\boxed{A}$

(18)

g' +   +   0   −   −   −   0   +   +

−4   −3   −2   −1   0   1   2   3   4

g decreasing $-2 \le x \le 2$ $\boxed{A}$

(19) slope → $2x + 3$

original equation? $\int 2x + 3 \, dx$

$y = x^2 + 3x + C$

check C, or D using pt. (1,2)

(C) $2 = 1^2 + 3(1)$

$2 = 1 + 3$

$2 \neq 4$

(D) $2 = 1^2 + 3(1) - 2$

$2 = 1 + 3 - 2$

$2 = 2$ ✓

$\boxed{D}$

(20) I. $\lim_{x \to 3} f(x)$ exists → true if $\lim_{x \to 3^-} = \lim_{x \to 3^+}$

$\lim_{x \to 3^-} x + 2 = \lim_{x \to 3} 4x - 7$

$3 + 2 = 4(3) - 7$

$5 \overset{\checkmark}{=} 5$

II. f cont. @ $x = 3$ → true if $\lim_{x \to 3} f(x) = f(3)$

$5 = 3 + 2$

$5 \overset{\checkmark}{=} 5$

$\boxed{D}$ I & II only!

III. f diff'able @ $x = 3$, true if $\overset{\text{left}}{f'(3)} = \overset{\text{right}}{f'(3)}$

$1 \neq 4$ false.

(21) looking at 2nd derivative – concavity

pt. of inf → when f changes concavity

f'' (+)   (−)   (+)   (+)

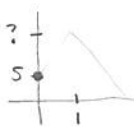
a   0   b

pt. of inf @ a and 0. $\boxed{A}$

(22)

f' +   −

1



(a) too small

(b) too small

(c) 6 → if $f'(x) = 0$ $0 \le x \le 1$; it doesn't!

(d) 8 → since $f'(x) > 0$ for $0 \le x \le 1$

(e) 11 → if $f'(x) = 6$, $0 \le x \le 1$; it doesn't!

OR

$\int f'(x) = \int -6x + 6$

$f(x) = -3x^2 + 6x + C$

$f(0) = -3(0)^2 + 6(0) + C$

$5 = C$

$\boxed{D}$

$f(x) = -3x^2 + 6x + 5$

$f(1) = -3 + 6 + 5$

$f(1) = 8$

(23) $\frac{d}{dx}\int_0^{x^2} \sin(t^3)\,dt$

by 2nd FTC, chain rule

$= \sin(x^2)^3 \cdot 2x$

$= 2x \sin(x^6)$ $\boxed{E}$

(24) $f(x) = 4x^3 - 5x + 3$ tangent line – need slope + pt.

$f'(x) = 12x^2 - 5$

$f'(-1) = 12(-1)^2 - 5$

$= 12 - 5$

$= 7 \rightarrow$ slope

$f(-1) = 4(-1)^3 - 5(-1) + 3$

$= -4 + 5 + 3$

$= 4 \rightarrow$ pt. $(-1, 4)$

$y - y_1 = m(x - x_1)$

$y - 4 = 7(x + 1)$

$y = 7x + 11$ $\boxed{C}$

(25) $x(t) = 2t^3 - 21t^2 + 72t - 53$ position

$x'(t) = v(t) \rightarrow$ velocity when $v(t) = 0$, particle is resting.

$v(t) = 6t^2 - 42t + 72$

$0 = 6(t^2 - 7t + 12)$

$0 = 6(t-3)(t-4)$

$\cancel{6=0}$ $t - 3 = 0$ $t - 4 = 0$

$t = 3$ $t = 4$ $\boxed{E}$

(26) slope $\rightarrow$ derivative!

$3y^2 - 2x^2 = 6 - 2xy$ @ $(3,2)$

$6y\frac{dy}{dx} - 4x = -2x\frac{dy}{dx} + y(-2)$

$6(2)\frac{dy}{dx} - 4(3) = -2(3)\frac{dy}{dx} + (2)(-2)$

$12\frac{dy}{dx} - 12 = -6\frac{dy}{dx} - 4$

$18\frac{dy}{dx} = 8$

$\frac{dy}{dx} = \frac{8}{18} \Rightarrow \frac{4}{9}$ $\boxed{B}$

(27) $f(x) = x^3 + x$

$f'(x) = 3x^2 + 1$

$f'(1) = 3 + 1$

$= 4$

$g(x) = f^{-1}(x)$

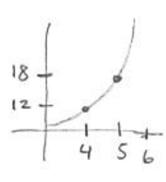
$g'(x) = \frac{1}{f'(y)}$

$g'(2) = \frac{1}{f'(1)}$

$g'(2) = \frac{1}{4}$ $\boxed{B}$

(28) $g'(x) > 0 \Rightarrow g$ increasing

$g''(x) > 0 \Rightarrow g$ concave up



a) 15 too small

b) 18 too small

c) 21 inc but not high enough

d) 24 inc by 6, would make a straight line

e) 27 ✓ $\boxed{E}$