t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

$\Delta t = 0.8$ $\Delta t = 0.8$ $\Delta t = 0.8$

M M M

1. Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

(a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure. $\hookrightarrow a(t) = v'(t) = \frac{\Delta v}{\Delta t}$

$$a(1.4) \approx \frac{v(1.6) - v(1.2)}{1.6 - 1.2} \quad \frac{\to \text{miles/hr}}{\to \text{hr}} \ldots ☺$$

$$\approx \frac{16.3 - 17.2}{1.6 - 1.2}$$

$$\approx -2.25 \text{ miles/hr}^2$$

1pt - approx.

1pt - units

(b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

$n = 3$ $\hookrightarrow \Delta x = \frac{2.4 - 0}{3} = 0.8$

$$\int_0^{2.4} v(t)dt \approx 0.8(11.8) + 0.8(17.2) + 0.8(16.8) \quad \to (\text{hr})(\text{miles/hr}) \ldots ☺$$

$$\approx 36.64 \text{ miles}$$

1pt - mdpt sum

1pt - approx.

$\int_0^{2.4} v(t)dt$ is the displacement of Ruth, in miles from $t = 0$ to $t = 2.4$ hrs.

1pt - interpretation w/ units

~~It is ...~~

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Continue problem 1 on page 5.

function is velocity

(c) For $0 \le t \le 2.4$ hours, Ruth's velocity can be modeled by the function g given by $\boxed{g(t) = \frac{24t + 5\sin(6t)}{t + 0.7}}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 2.4$?

avg function that is velocity $= \frac{1}{b-a} \int$ function dt

avg velocity $= \frac{1}{2.4-0} \int_0^{2.4} g(t)\,dt$

$= 14.064$ miles/hr

1pt - integral

1pt - answer

OR

where $x(t)$ is position of Ruth

avg velocity $= \frac{x(2.4)-x(0)}{2.4-0}$

$= \frac{1}{2.4-0}\left(x(2.4)-x(0)\right)$

$= \frac{1}{2.4}\left(\int_0^{2.4} g(t)\,dt\right)$

$= 14.064$

$\int_0^{2.4} g(t)\,dt = x(t)\Big|_0^{2.4}$

$= x(2.4)-x(0)$

☺

(d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$? Give a reason for your answer.

use $g(t)$ as $v(t)$

$v(t)$ & $a(t)$ same signs

$v(t)$ & $a(t)$ different signs

velocity → $g(1.3) = 18.096 > 0$

acceleration → $g'(1.3) = 3.761 > 0$

Ruth's speed is inc @ $t = 1.3$ b/c $g(1.3) > 0$ and $g'(1.3) > 0$

both velocity + acceleration have the same signs

2pts - conclusion w/ reason

GO ON TO THE NEXT PAGE.

2. A store is having a 12-hour sale. The total number of shoppers who have entered the store t hours after the sale begins is modeled by the function S defined by $S(t) = 0.5t^4 - 16t^3 + 144t^2$ for $0 \le t \le 12$. At time $t = 0$, when the sale begins, there are no shoppers in the store.

initial

(a) At what rate are shoppers entering the store 3 hours after the start of the sale?

$S'(3)$

$S'(3) = 486$ shoppers/hr

1pt: answer

(b) Find the value of $\frac{1}{3}\int_6^9 S'(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{3}\int_6^9 S'(t)\,dt$ in the context of this problem.

$\frac{1}{3}\int_6^9 S'(t)\,dt = \frac{1}{3} S(t)\Big|_6^9$

shoppers

$= \frac{1}{3}(S(9) - S(6))$

hr

$= 301.5$ shoppers/hr

1pt: value of $\frac{1}{3}\int_6^9 S'(t)\,dt$

$\frac{1}{3}\int_6^9 S'(t)\,dt$ is the average rate at which shoppers enter store, in shoppers/hr, from $t=6$ and $t=9$ hours.

1pt: meaning of $\frac{1}{3}\int_6^9 S'(t)\,dt$

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Continue problem 2 on page 7.

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(c) The rate at which shoppers leave the store, measured in shoppers per hour, is modeled by the function L defined by $\boxed{L(t) = -80 + \frac{4400}{t^2 - 14t + 55}}$ for $0 \le t \le 12$. According to the model, how many shoppers are in the store at the end of the sale (time $t = 12$)? Give your answer to the nearest whole number.

shoppers in store = initial + $\cancel{\int_0^{12} \text{rate enter}}$ + # enter store $- \int_0^{12}$ rate leave

$= 0 + S(12) - \int_0^{12} L(t)\,dt$

$= 195.702$

1pt: integral
1pt: uses $S(12)$
1pt: answer

(d) Using the given models, find the time t, $0 \le t \le 12$, at which the number of shoppers in the store is the greatest. Justify your answer.

abs. max

shoppers $= S(t) - \int_0^t L(x)\,dx$

rate shoppers $= S'(t) - L(t)$

$S'(t) - L(t) = 0$

$S'(t) = L(t)$

$t = 5.545$

1pt: considers $S'(t) - L(t) = 0$

shoppers @ $t = 0$, 0

@ $t = 12$, 195.702

@ $t = 5.545$, 1361.833

1pt: answer
1pt: reason

$\therefore$, # shoppers in store greatest @ time, $t = 5.545$ hrs.

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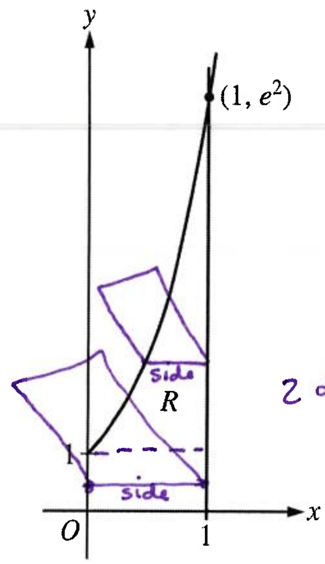

2 different squares ... 2 different sides

3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$.

$y - y_1 = m(x - x_1)$

$f(1) = e^2$

$f'(x) = 2 \cdot e^{2x}$

$f'(1) = 2e^2$ 1pt: $f'(1)$

$\boxed{y - e^2 = 2e^2(x-1)}$

1pt: answer

Continue problem 3 on page 15.

(b) Find the area of R.

Area of R $= \int_0^1 e^{2x}\,dx$

$u = 2x$, $u(1)=2$, $u(0)=0$
$du = 2dx$
$\frac{1}{2}du = dx$

$= \frac{1}{2}\int_0^2 e^u\,du$

$= \frac{1}{2}e^u\Big|_0^2$

$= \frac{1}{2}(e^2 - e^0)$ ← ok to stop here

$= \frac{1}{2}(e^2 - 1)$

1pt: integral
1pt: antiderivative
1pt: answer

(c) Region R forms the base of a solid whose cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

→ in terms of y

① square

② Area = $(\text{side})^2$

$= (1 - \frac{1}{2}\ln y)^2$

$y = e^{2x}$
$\ln y = 2x$
$\frac{1}{2}\ln y = x$

③ Volume $= \int_0^1 1^2\,dy + \int_1^{e^2} (1 - \frac{1}{2}\ln y)^2\,dy$

2pts: integrand
1pt: limits
1pt: answer

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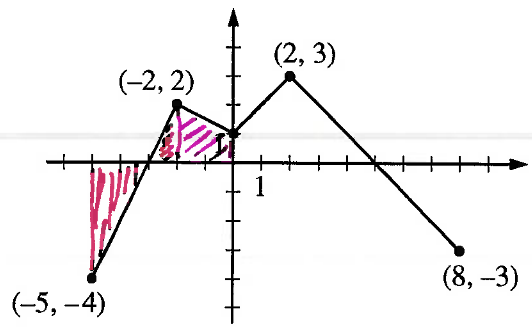

Graph of f

4. The continuous function f is defined on the interval $-5 \le x \le 8$. The graph of f, which consists of four line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(-5)$.

$g(0) = 2(0) + \int_{-2}^{0} f(t)\,dt$

$= 0 + \frac{1}{2}(2)(1) + (2)(1)$ ← ok to stop here for 1pt

$= 3$

1pt: g(0)

$g(-5) = 2(-5) + \int_{-2}^{-5} f(t)\,dt$

$= -10 - \int_{-5}^{-2} f(t)\,dt$

$= -10 - \left(\frac{1}{2}(2)(-4) + \frac{1}{2}(1)(2)\right)$

$= -10 - (-4+1)$

$= -7$

1pt: g(−5)

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

$g(x) = 2x + \int_{-2}^{x} f(t)\,dt$

$g'(x) = 2 + f(x)$

$g''(x) = f'(x)$

$g''(4) = f'(4)$

$= \frac{8-2}{-3-3}$

$\boxed{g''(4) = -1}$

$g''(-2) = f'(-2)$

$\boxed{g''(-2) \text{ DNE}}$ (b/c $\lim_{x\to -2^-} f'(x) \neq \lim_{x\to -2^+} f'(x)$) ☺

1pt: g'(x)
1pt: g''(4)
1pt: g''(−2)

Continue problem 4 on page 17.

(c) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

$\to g'' < 0$? or g' dec?

$g''(x) = \underbrace{f'(x) < 0}_{f \text{ dec}}$

g is concave down on $(-2, 0) \cup (2, 8)$

b/c f dec on $(-2, 0) \cup (2, 8)$

OR b/c g' dec on $(-2, 0) \cup (2, 8)$

OR b/c $g'' < 0$ on $(-2, 0) \cup (2, 8)$

1 pt: intervals w/ reason

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(d) The function h is given by $h(x) = g(x^3 + 1)$. Find $h'(1)$. Show the work that leads to your answer.

$h'(x) = 3x^2 \cdot g'(x^3+1)$

2pts: chain rule

$h'(1) = 3(1)^2 \cdot g'(1^3+1)$

$= 3 \cdot g'(2)$

$= 3 \cdot (2 + f(2))$

1 pt: answer

$= 3(2 + 3)$ ← ok to stop here

$= 15$

5. Particle X moves along the positive x-axis so that its position at time $t \geq 0$ is given by $x(t) = 5t^3 - 9t^2 + 7$.

(a) Is particle X moving toward the left or toward the right at time $t = 1$? Give a reason for your answer.

↳ $v(t) < 0$ ↳ $v(t) > 0$

$x'(t) = 15t^2 - 18t$ 1pt : considers $x'(1)$

$x'(1) = 15 - 18$

$= -3 < 0$

The particle is moving left @ $t = 1$ b/c $x'(1) < 0$ 1pt : answer w/ reason

(b) At what time $t \geq 0$ is particle X farthest to the left? Justify your answer.

↳ abs. min

$x'(t) = 15t^2 - 18t$

$0 = 3t(5t - 6)$ 1pt : considers $x'(t) = 0$

$t = 0, t = 6/5$

x': − + ; 0 (1) 6/5 (10) 1pt : IDs $t = 6/5$

Particle farthest left @ $t = 6/5$

b/c $x'(t) < 0$ on $(0, 6/5)$ and $x'(t) > 0$ on $(6/5, \infty)$ 1pt: answer w/ reason

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Continue problem 5 on page 19.

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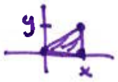
(c) A second particle, Y, moves along the positive y-axis so that its position at time t is given by $y(t) = 7t + 3$. At any time t, $t \geq 0$, the origin and the positions of the particles X and Y are the vertices of a triangle in the first quadrant. Find the rate of change of the area of the triangle at time $t = 1$. Show the work that leads to your answer.

$\frac{dA}{dt} = ?$



$$\text{Area of } \Delta = \frac{1}{2}(\text{base})(\text{height})$$
$$= \frac{1}{2}\, x(t) \cdot y(t)$$
$$A(t) = \frac{1}{2}(5t^3 - 9t^2 + 7)(7t + 3)$$

1pt: area function

$$A'(t) = \frac{1}{2}\left[(7t+3)(15t^2 - 18t) + (5t^3 - 9t^2 + 7)(7)\right]$$

2pts: derivative

$$A'(1) = \frac{1}{2}\left[(7+3)(15-18) + (5-9+7)(7)\right]$$

← ok to stop here for AP

$$= \frac{1}{2}\left[10(-3) + (3)(7)\right]$$
$$= -\frac{9}{2}$$

1pt: answer

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

6. Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$, where $x \neq 0$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(1) = 2$.

(a) Find the slope of the line tangent to the graph of f at the point $(1, 2)$.

↳ dy/dx

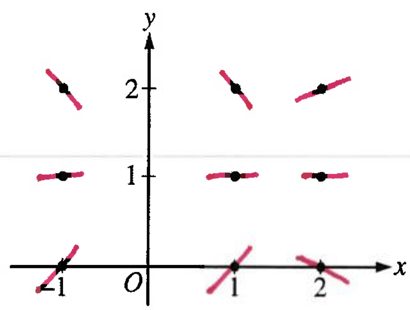
$$\left.\frac{dy}{dx}\right|_{(1,2)} = \left(1 - \frac{2}{1^2}\right)(2-1)$$

$$= -1$$

1 pt: answer

(b) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

y \ x	-1	1	2
2	-1	-1	½
1	0	0	0
0	1	1	-½



1 pt: zero slopes

1 pt: remaining slopes

Continue problem 6 on page 21.

NO CALCULATOR ALLOWED

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$ with initial condition $f(1) = 2$.

solve diff'eq' by separate variables
x w/dx, y w/dy

$$\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y-1)$$

$$dy = \left(1 - \frac{2}{x^2}\right)(y-1)\,dx$$

$$\int \frac{1}{y-1}\,dy = \int \left(1 - \frac{2}{x^2}\right)dx$$ → 1pt: separate variables

$u = y-1$
$du = dy$

$$\int \frac{1}{u}\,du = \int (1 - 2x^{-2})\,dx$$ → 2pts: antiderivative, 1pt: "+C"

$$\ln|u| = x + 2x^{-1} + C$$

$$\ln|y-1| = x + \frac{2}{x} + C$$ ← initial condition (1,2)

$$\ln|2-1| = 1 + \frac{2}{1} + C$$ ← 1pt: use initial condition

$$\ln 1 = 3 + C$$

$$0 = 3 + C$$

$$-3 = C$$

pos. b/c $y-1>0$, $2-1>0$, $1>0$ ...☺

$$\ln|y-1| = x + \frac{2}{x} - 3$$

$$|y-1| = e^{x + \frac{2}{x} - 3}$$

$$y - 1 = e^{x + \frac{2}{x} - 3}$$

$$\boxed{y = e^{x + \frac{2}{x} - 3} + 1}$$

1pt: solves for y.

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