CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1)   Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2)   The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. $\int \left(5e^{2x} + \dfrac{1}{x} \right) dx =$

(A) $\dfrac{5}{2}e^{2x} + \dfrac{2}{x^2} + C$

(B) $\dfrac{5}{2}e^{2x} + \ln|x| + C$

(C) $5e^{2x} + \dfrac{2}{x^2} + C$

(D) $5e^{2x} + \ln|x| + C$

(E) $10e^{2x} - \dfrac{1}{x^2} + C$

2. If $f(x) = \sqrt{x} + \dfrac{3}{\sqrt{x}}$, then $f'(4) =$

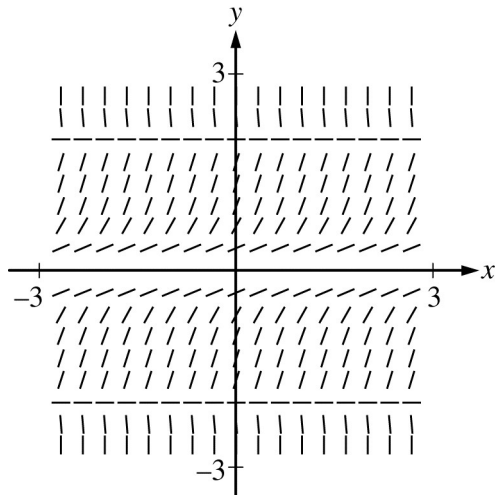(A) $\dfrac{1}{16}$ (B) $\dfrac{5}{16}$ (C) 1 (D) $\dfrac{7}{2}$ (E) $\dfrac{49}{4}$

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3. $\int x^2 \left(x^3 + 5\right)^6 dx =$

(A) $\frac{1}{3}\left(x^3 + 5\right)^6 + C$

(B) $\frac{1}{3}x^3 \left(\frac{1}{4}x^4 + 5x\right)^6 + C$

(C) $\frac{1}{7}\left(x^3 + 5\right)^7 + C$

(D) $\frac{3}{7}x^2 \left(x^3 + 5\right)^7 + C$

(E) $\frac{1}{21}\left(x^3 + 5\right)^7 + C$

x	0	25	30	50
$f(x)$	4	6	8	12

4. The values of a continuous function f for selected values of x are given in the table above. What is the value of the left Riemann sum approximation to $\int_0^{50} f(x)\,dx$ using the subintervals $[0, 25]$, $[25, 30]$, and $[30, 50]$?

   (A) 290   (B) 360   (C) 380   (D) 390   (E) 430

$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^2 + cx - 18 & \text{for } x \geq 2 \end{cases}$$

5. Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

   (A) 2   (B) 7   (C) 9   (D) $4\pi - 4$ (E) There is no such value of c.

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6. Which of the following is an antiderivative of $3\sec^2 x + 2$?

   (A) $3\tan x$ (B) $3\tan x + 2x$ (C) $3\sec x + 2x$ (D) $\sec^3 x + 2x$ (E) $6\sec^2 x \tan x$

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7. If $f(x) = x^2 - 4$ and g is a differentiable function of x, what is the derivative of $f(g(x))$?

   (A) $2g(x)$ (B) $2g'(x)$ (C) $2xg'(x)$ (D) $2g(x)g'(x)$ (E) $2g(x) - 4$

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8. Shown above is a slope field for the differential equation $\dfrac{dy}{dx} = y^2\left(4 - y^2\right)$. If $y = g(x)$ is the solution to the differential equation with the initial condition $g(-2) = -1$, then $\lim\limits_{x \to \infty} g(x)$ is

(A) $-\infty$ (B) -2 (C) 0 (D) 2 (E) 3

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9. If $f''(x) = x(x + 2)^2$, then the graph of f is concave up for

(A) $x < 0$

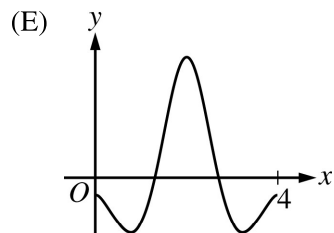(B) $x > 0$

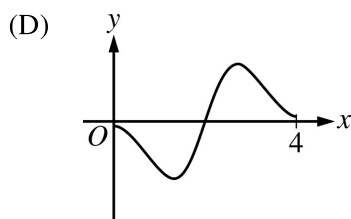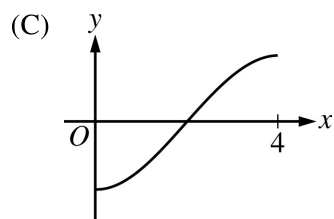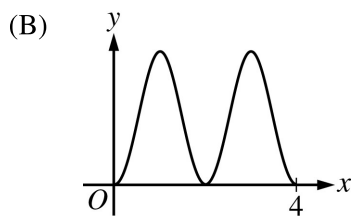(C) $-2 < x < 0$

(D) $x < -2$ and $x > 0$

(E) all real numbers

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10. If $y = \sin x \cos x,$ then at $x = \dfrac{\pi}{3},$ $\dfrac{dy}{dx} =$

    (A) $-\dfrac{1}{2}$ (B) $-\dfrac{1}{4}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{2}$ (E) 1

11. $\displaystyle\lim_{x\to-3} \dfrac{x^2 - 9}{x^2 - 2x - 15}$ is

    (A) 0   (B) $\dfrac{3}{5}$ (C) $\dfrac{3}{4}$ (D) 1   (E) nonexistent

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12. What is the average rate of change of $y = \cos(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$?

(A) $-\dfrac{4}{\pi}$ (B) -1 (C) 0 (D) $\dfrac{\sqrt{2}}{2}$ (E) $\dfrac{4}{\pi}$

13. If $y^3 + y = x^2$, then $\dfrac{dy}{dx} =$

(A) 0 (B) $\dfrac{x}{2}$ (C) $\dfrac{2x}{3y^2}$ (D) $2x - 3y^2$ (E) $\dfrac{2x}{1 + 3y^2}$

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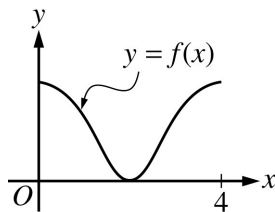
14. The graph of $y = f(x)$ on the closed interval $[0, 4]$ is shown above. Which of the following could be the graph of $y = f'(x)$?

(A)

(B)

(C)

(D)

(E)

GO ON TO THE NEXT PAGE.

$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 1 \\ \ln(3x - 2) & \text{if } x \geq 1 \end{cases}$$

15. Let f be the function defined above. Which of the following statements about f are true?

    I. $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x)$

    II. $\lim\limits_{x \to 1^-} f'(x) = \lim\limits_{x \to 1^+} f'(x)$

    III. f is differentiable at $x = 1$.

    (A) None

    (B) I only

    (C) II only

    (D) II and III only

    (E) I, II, and III

16. The function f is defined by $f(x) = 2x^3 - 4x^2 + 1$. The application of the Mean Value Theorem to f on the interval $1 \leq x \leq 3$ guarantees the existence of a value c, where $1 < c < 3$, such that $f'(c) =$

    (A) 0   (B) 9   (C) 10   (D) 14   (E) 16

17. The velocity v, in meters per second, of a certain type of wave is given by $v(h) = 3\sqrt{h}$, where h is the depth, in meters, of the water through which the wave moves. What is the rate of change, in meters per second per meter, of the velocity of the wave with respect to the depth of the water, when the depth is 2 meters?

    (A) $-\dfrac{3}{4\sqrt{2}}$ (B) $-\dfrac{3}{8\sqrt{2}}$ (C) $\dfrac{3}{2\sqrt{2}}$ (D) $\dfrac{3}{\sqrt{2}}$ (E) $4\sqrt{2}$

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18. If $\dfrac{dy}{dt} = -10e^{-t/2}$ and $y(0) = 20,$ what is the value of $y(6)?$

    (A) $20e^{-6}$ (B) $20e^{-3}$ (C) $20e^{-2}$ (D) $10e^{-3}$ (E) $5e^{-3}$

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19. Let f be the function with derivative defined by $f'(x) = x^3 - 4x.$ At which of the following values of x does the graph of f have a point of inflection?

    (A) 0   (B) $\dfrac{2}{3}$ (C) $\dfrac{2}{\sqrt{3}}$ (D) $\dfrac{4}{3}$ (E) 2

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20. Let f be the function given by $f(x) = \dfrac{(x-4)(2x-3)}{(x-1)^2}$. If the line $y = b$ is a horizontal asymptote to the graph of f, then $b =$

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

21. The base of a solid is the region bounded by the x-axis and the graph of $y = \sqrt{1-x^2}$. For the solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) $\dfrac{2}{3}$ (B) $\dfrac{4}{3}$ (C) 2   (D) $\dfrac{2\pi}{3}$ (E) $\dfrac{4\pi}{3}$

22. Let f be the function given by $f(x) = \dfrac{kx}{x^2 + 1}$, where k is a constant. For what values of k, if any, is f strictly decreasing on the interval $(-1, 1)$?

(A) $k < 0$

(B) $k = 0$

(C) $k > 0$

(D) $k > 1$ only

(E) There are no such values of k.

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23. Which of the following is an equation for the line tangent to the graph of $y = 3 - \int_{-1}^{x} e^{-t^3}\, dt$ at the point where $x = -1$?

(A) $y - 3 = -3e(x + 1)$

(B) $y - 3 = -e(x + 1)$

(C) $y - 3 = 0$

(D) $y - 3 = \dfrac{1}{e}(x + 1)$

(E) $y - 3 = 3e(x + 1)$

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24. Which of the following is the solution to the differential equation $\dfrac{dy}{dx} = 5y^2$ with the initial condition $y(0) = 3$?

(A) $y = \sqrt{9e^{5x}}$

(B) $y = \sqrt{\dfrac{1}{9}e^{5x}}$

(C) $y = \sqrt{e^{5x} + 9}$

(D) $y = \dfrac{3}{1 - 15x}$

(E) $y = \dfrac{3}{1 + 15x}$

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25. $\lim\limits_{h \to 0} \dfrac{\sin\left(\dfrac{\pi}{3} + h\right) - \sin\left(\dfrac{\pi}{3}\right)}{h}$ is

(A) 0   (B) $\dfrac{1}{2}$ (C) 1   (D) $\dfrac{\sqrt{3}}{2}$ (E) nonexistent

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26. An object moves along a straight line so that at any time $t \geq 0$ its velocity is given by $v(t) = 2\cos(3t)$. What is the distance traveled by the object from $t = 0$ to the first time that it stops?

(A) 0   (B) $\dfrac{\pi}{6}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{\pi}{3}$ (E) $\dfrac{4}{3}$

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x	$f(x)$	$f'(x)$
0	49	0
1	2	−8
2	−1	−80

27. The table above gives selected values for a differentiable and decreasing function f and its derivative. If f^{-1} is the inverse function of f, what is the value of $\left(f^{-1}\right)'(2)$?

(A) -80 (B) $-\dfrac{1}{8}$ (C) $-\dfrac{1}{80}$ (D) $\dfrac{1}{80}$ (E) $\dfrac{1}{8}$

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28. The top of a 15-foot-long ladder rests against a vertical wall with the bottom of the ladder on level ground, as shown above. The ladder is sliding down the wall at a constant rate of 2 feet per second. At what rate, in radians per second, is the acute angle between the bottom of the ladder and the ground changing at the instant the bottom of the ladder is 9 feet from the base of the wall?

(A) $-\dfrac{2}{9}$ (B) $-\dfrac{1}{6}$ (C) $-\dfrac{2}{25}$ (D) $\dfrac{2}{25}$ (E) $\dfrac{1}{9}$

END OF PART A OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

PART B STARTS ON PAGE 24.

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1)   The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2)   Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3)   The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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76. The function $P(t)$ models the population of the world, in billions of people, where t is the number of years since January 1, 2010. Which of the following is the best interpretation of the statement $P'(1) = 0.076$?

(A) On February 1, 2010, the population of the world was increasing at a rate of 0.076 billion people per year.

(B) On January 1, 2011, the population of the world was increasing at a rate of 0.076 billion people per year.

(C) On January 1, 2011, the population of the world was 0.076 billion people.

(D) From January 1, 2010 to January 1, 2011, the population of the world was increasing at an average rate of 0.076 billion people per year.

(E) When the population of the world was 1 billion people, the population of the world was increasing at a rate of 0.076 billion people per year.

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x	0	2	4	6	8	10
$f(x)$	5	7	8	0	−15	−20

77. Let f be a differentiable function with selected values given in the table above. What is the average rate of change of f over the closed interval $0 \leq x \leq 10$?

(A) −6   (B) $-\dfrac{5}{2}$ (C) −2   (D) $-\dfrac{2}{5}$ (E) $\dfrac{2}{5}$

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78. The rate at which motor oil is leaking from an automobile is modeled by the function L defined by $L(t) = 1 + \sin\left(t^2\right)$ for time $t \geq 0$. $L(t)$ is measured in liters per hour, and t is measured in hours. How much oil leaks out of the automobile during the first half hour?

(A) 1.998 liters

(B) 1.247 liters

(C) 0.969 liters

(D) 0.541 liters

(E) 0.531 liters

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	4	2	π

79. The table above gives values of the differentiable functions f and g and their derivatives at $x = 0$.

If $h(x) = \dfrac{f(x)}{g(x)}$, what is the value of $h'(0)$?

(A) $\dfrac{8 - 3\pi}{4}$ (B) $\dfrac{3\pi - 8}{4}$ (C) $\dfrac{4}{\pi}$ (D) $\dfrac{2 - 3\pi}{2}$ (E) $\dfrac{8 + 3\pi}{4}$

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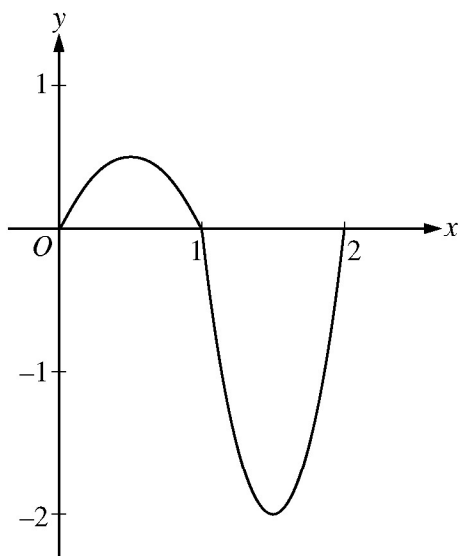
Graph of f'

80. The figure above shows the graph of f', the derivative of a function f, for $0 \leq x \leq 2$. What is the value of x at which the absolute minimum of f occurs?

(A) 0   (B) $\frac{1}{2}$ (C) 1   (D) $\frac{3}{2}$ (E) 2

81. What is the area of the region enclosed by the graphs of $y = \sqrt{4x - x^2}$ and $y = \frac{x}{2}$?

(A) 1.707   (B) 2.829   (C) 5.389   (D) 8.886   (E) 21.447

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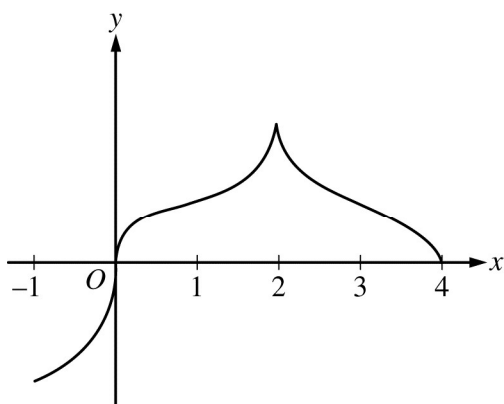
Graph of f'

82. The graph of f', the derivative of f, is shown above. The line tangent to the graph of f' at $x = 0$ is vertical, and f' is not differentiable at $x = 2$. Which of the following statements is true?

(A) f' does not exist at $x = 2$.

(B) f is decreasing on the interval $(2, 4)$.

(C) The graph of f has a point of inflection at $x = 2$.

(D) The graph of f has a point of inflection at $x = 0$.

(E) f has a local maximum at $x = 0$.

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83. A particle moves along the *x*-axis so that its position at time $t > 0$ is given by $x(t)$ and

    $$\frac{dx}{dt} = -10t^4 + 9t^2 + 8t.$$ The acceleration of the particle is zero when $t =$

    (A) 0.387   (B) 0.831   (C) 1.243   (D) 1.647   (E) 8.094

84. The function f is continuous on the closed interval $[1, 7]$. If $\int_1^7 f(x)\, dx = 42$ and $\int_7^3 f(x)\, dx = -32,$

    then $\int_1^3 2f(x)\, dx =$

    (A) −148   (B) 10   (C) 12   (D) 20   (E) 148

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85. Let $y = f(x)$ define a twice-differentiable function and let $y = t(x)$ be the line tangent to the graph of f at $x = 2$. If $t(x) \geq f(x)$ for all real x, which of the following must be true?

(A) $f(2) \geq 0$

(B) $f'(2) \geq 0$

(C) $f'(2) \leq 0$

(D) $f''(2) \geq 0$

(E) $f''(2) \leq 0$

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86. The vertical line $x = 2$ is an asymptote for the graph of the function f. Which of the following statements must be false?

(A) $\lim\limits_{x \to 2} f(x) = 0$

(B) $\lim\limits_{x \to 2} f(x) = -\infty$

(C) $\lim\limits_{x \to 2} f(x) = \infty$

(D) $\lim\limits_{x \to \infty} f(x) = 2$

(E) $\lim\limits_{x \to \infty} f(x) = \infty$
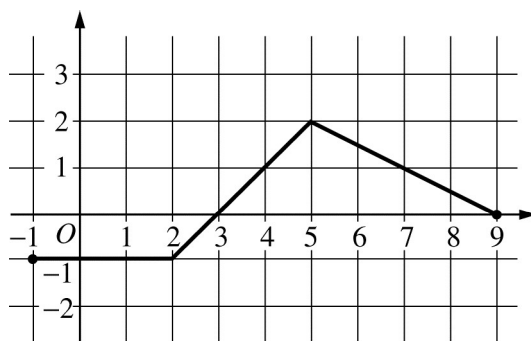
Graph of f

87. The graph of the piecewise linear function f is shown above. Let h be the function given by $h(x) = \int_{-1}^{x} f(t)\, dt$. On which of the following intervals is h increasing?

(A) $[-1, 3]$

(B) $[0, 5]$

(C) $[2, 5]$ only

(D) $[2, 9]$

(E) $[3, 9]$ only

GO ON TO THE NEXT PAGE.

88. The first derivative of the function f is given by $f'(x) = \sin\left(x^2\right)$. At which of the following values of x does f have a local minimum?

(A) 2.507   (B) 2.171   (C) 1.772   (D) 1.253   (E) 0

89. If $\lim\limits_{x \to a} f(x) = f(a)$, then which of the following statements about f must be true?

(A) f is continuous at $x = a$.

(B) f is differentiable at $x = a$.

(C) For all values of x, $f(x) = f(a)$.

(D) The line $y = f(a)$ is tangent to the graph of f at $x = a$.

(E) The line $x = a$ is a vertical asymptote of the graph of f.

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90. The temperature F, in degrees Fahrenheit $(°F)$, of a cup of coffee t minutes after it is poured is given by $F(t) = 72 + 118e^{-0.093t}$. To the nearest degree, what is the average temperature of the coffee between $t = 0$ and $t = 10$ minutes?

    (A)   93°F

    (B) 119°F

    (C) 146°F

    (D) 149°F

    (E) 154°F

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91. If $f'(x) = \cos\left(x^2\right)$ and $f(3) = 7$, then $f(2) =$

    (A) 0.241   (B) 5.831   (C) 6.416   (D) 6.759   (E) 7.241

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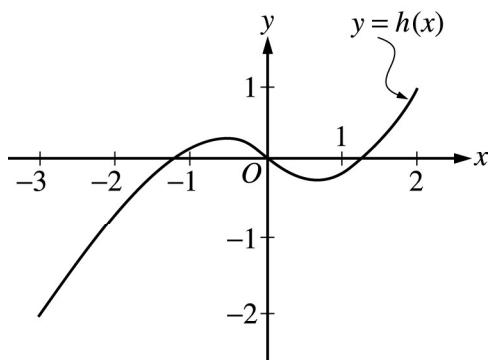
92. The graph of the function h is shown in the figure above. Of the following, which has the greatest value?

(A) Average value of h over $[-3,2]$

(B) Average rate of change of h over $[-3,2]$

(C) $\int_{-3}^{2} h(x)\,dx$

(D) $\int_{-3}^{0} h(x)\,dx$

(E) $h'(0)$