CALCULUS AB FINAL EXAM SEMESTER 1 REVIEW

PART I NON-CALCULATOR: MULTIPLE-CHOICE
NO calculator may be used on this part of the review.

1. f is continuous for $a \le x \le b$ but not differentiable for some c such that $a < c < b$. Which of the following could be true?

(A) $x = c$ is a vertical asymptote of the graph of f. f discont @ x=c b/c asymptote so not true

(B) $\lim_{x \to c} f(x) \neq f(c)$ means f discont @ x=c b/c hole, so not true

$\boxed{\text{(C)}}$ The graph of f has a cusp at $x = c$. f not diff'able x=c but cont @ x=c

(D) $f(c)$ is undefined. means f discont @ x=c b/c hole, so not true

(E) None of the above

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2. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

$$6x + y(2) + 2x\left(\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -6x - 2y$$

$$\frac{dy}{dx}(2x + 2y) = -6x - 2y$$

$$\frac{dy}{dx} = \frac{-2(3x+y)}{2(x+y)}$$

$$\frac{dy}{dx} = -\frac{3x+y}{x+y}$$

(A) $-\frac{3x+2y}{y^2}$

$\boxed{\text{(B)}}$ $-\frac{3x+y}{x+y}$

(C) $\frac{1-3x-y}{x+y}$

(D) $-\frac{3x}{1+y}$

(E) $-\frac{3x}{x+y}$

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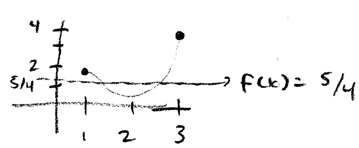
x	1	2	3
$f(x)$	2	k	4

3. The function f is continuous on the closed interval [1,3] and has the values in the table given above. The equation $f(x) = \frac{5}{4}$ must have at least two solutions in the interval [1,3] if $k =$

IVT



f(2) must be less than 5/4 in order for f(x) = 5/4 to have two solutions

$\boxed{\text{(A)}}$ $\frac{1}{4}$

(B) $\frac{3}{2}$

(C) 2

(D) $\frac{9}{4}$

(E) 3

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4. If $\frac{d}{dx}f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx}f(h(x)) =$ $f'(h(x)) \cdot h'(x)$ chain rule

$$= f'(x^2) \cdot 2x$$

$$= g(x^2) \cdot 2x$$

$$= 2x\, g(x^2)$$

(A) $g(x^2)$

(B) $2xg(x)$

(C) $g'(x)$

$\boxed{\text{(D)}}$ $2xg(x^2)$

(E) $x^2g(x^2)$

5. If $f(x) = \tan 3x$, then $f'\left(\frac{\pi}{9}\right) =$

(A) $\frac{4}{3}$

(B) 4

(C) 6

$\boxed{\text{(D)}}$ 12

(E) $6\sqrt{3}$

$f'(x) = \sec^2(3x) \cdot 3$

$f'(\frac{\pi}{9}) = 3\sec^2(3 \cdot \frac{\pi}{9})$

$= 3(\sec\frac{\pi}{3})^2$

$= 3(2)^2 = 12$

$\cos\pi/3 = \frac{1}{2}$ ☺

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6. The function f is given by $f(x) = -x^6 + x^3 - 2$. On which of the following intervals is f decreasing?

(A) $(-\infty, 0)$

(B) $\left(-\infty, -\sqrt[3]{\frac{1}{2}}\right)$

(C) $\left(0, \sqrt[3]{\frac{1}{2}}\right)$

(D) $(0, \infty)$

$\boxed{\text{(E)}}$ $\left(+\sqrt[3]{\frac{1}{2}}, \infty\right)$

$f'(x) = -6x^5 + 3x^2$

$0 = 3x^2(-2x^3 + 1)$

$x = 0, \quad x = \sqrt[3]{\frac{1}{2}}$

f': $+$ $+$ $-$

-10 $\quad 0 \quad$ $.1$ $\quad \sqrt[3]{1/2} \quad$ 1

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7. An equation of the line tangent to the graph of $y = 3x - \cos x$ at $x = 0$ is

(A) $2x - y = 0$

(B) $2x - y = 1$

(C) $3x - y = -1$

$\boxed{\text{(D)}}$ $3x - y = 1$

(E) $4x - y = 0$

$y' = 3 + \sin x$

$y'(0) = 3 + \sin 0$

$= 3$

$y(0) = 3(0) - \cos 0$

$= -1$

$y - y_1 = m(x - x_1)$

$y - -1 = 3(x - 0)$

$y + 1 = 3x$

$1 = 3x - y$

$3x - y = 1$

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8. The graph of a twice-differentiable function f is shown below. Which of the following is true?

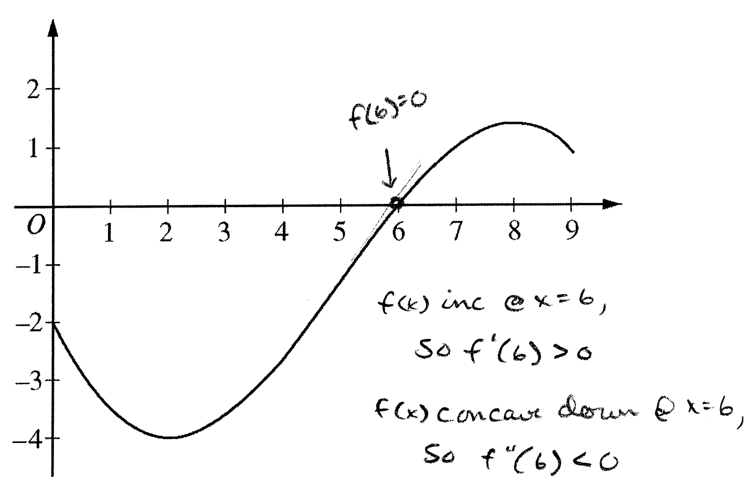
(A) $f(6) < f'(6) < f''(6)$

(B) $f(6) < f''(6) < f'(6)$

(C) $f'(6) < f(6) < f''(6)$

$\boxed{\text{(D)}}$ $f''(6) < f(6) < f'(6)$

(E) $f''(6) < f'(6) < f(6)$



Graph of f

$f''(6) < 0$, $\quad f(6) = 0$, $\quad f'(6) > 0$

$f''(6) < f(6) < f'(6)$

9. If $f(x) = \cos e^{2x}$, then $f'(x) =$

(A) $\sin e^{2x}$

(B) $2\sin e^{2x}$

(C) $-\sin e^{2x}$

(D) $-2\sin e^{2x}$

$\boxed{\text{(E)}}$ $-2e^{2x}\sin e^{2x}$

$f(x) = \cos(e^{2x})$

$f'(x) = -\sin(e^{2x}) \cdot e^{2x} \cdot 2$

$= -2e^{2x} \sin e^{2x}$

double chain ... ☺

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10. Determine the value of c so that *f(x)* continuous on the entire real line when $f(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$

(A) 0

(B) -2

(C) 1

$\boxed{\text{(D)}}$ $-\dfrac{1}{2}$

(E) None of these

$\lim_{x\to 2^-}(x+3) = \lim_{x\to 2^+}(cx+6)$

$2+3 = 2c+6$

$5 = 2c+6$

$-1 = 2c$

$-\frac{1}{2} = c$

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11. Find $\dfrac{dy}{dx}$ if: $x^2 + 3xy + y^3 = 10$

$\boxed{\text{(A)}}$ $-\dfrac{2x+3y}{3x+3y^2}$

(B) $\dfrac{2x-3y}{3x+3y^2}$

(C) $-\dfrac{x+y}{x+y^2}$

(D) $\dfrac{x-y}{x+y^2}$

(E) None of these

$2x + y(3) + 3x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$3x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -2x - 3y$

$\frac{dy}{dx} = \frac{-2x-3y}{3x+3y^2}$

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12. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top moving down the wall when the base of the ladder is 7 feet.

$\boxed{\text{(A)}}$ $\dfrac{7}{12}$ *ft / min*

(B) $-\dfrac{7}{12}$ *ft / min*

(C) 10π *ft / min*

(D) 9π *ft / min*

(E) None of these

$l = 25\text{ft}$

$\frac{dx}{dt} = 2\text{ft/sec}$

$x = 7\text{ft}$

$\frac{dy}{dt} = ?$

$x^2 + y^2 = l^2$

l is constant ... ☺

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(7)(2) + 2(24)\frac{dy}{dt} = 0$

$2(24)\frac{dy}{dt} = -2(7)(2)$

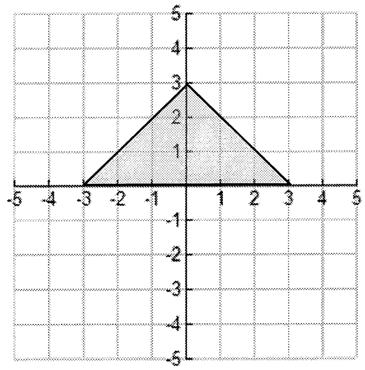
$\frac{dy}{dt} = \frac{-\cancel{2}(7)(2)}{\cancel{2}(24)}$

$= \frac{-7}{12}$ ft/sec

Ladder is moving $\frac{7}{12}$ ft/sec down the wall.

13. Set up a definite integral that yields the area of the region.

$f(x) = 3 - |x|$



$\int_a^b f(x)\,dx$ = area

$\int_{-3}^{3} (3-|x|)\,dx$ = area of shaded region

(A) $\int_3^3 (3 - x)\,dx$

(B) $\int_0^3 |x|\,dx$

(C) $\int_3^{-3} (3 - |x|)dx$

(D) $\boxed{}$ $\int_{-3}^{3} (3 - |x|)dx$

(E) $\int_3^0 |x|\,dx$

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14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 7t + 12$. For what value of t is the velocity of the particle zero?

↳ $v(t) = x'(t)$

$v(t) = 2t - 7$

$0 = 2t - 7$

$3.5 = t$

(A) 2.5

(B) 3

(C) 3.5

(D) 4

(E) 4.5

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15. If $f(x) = \sqrt{e^x}$, then $f'(\ln 2) =$

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) 1

(E) $\sqrt{2}$

$f(x) = (e^x)^{1/2}$

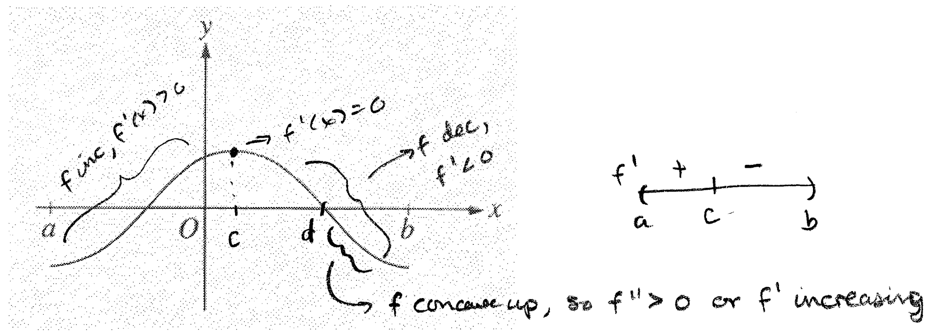
$f'(x) = \frac{1}{2}(e^x)^{-1/2} \cdot e^x$

$f'(\ln 2) = \frac{1}{2}(e^{\ln 2})^{-1/2} \cdot e^{\ln 2}$

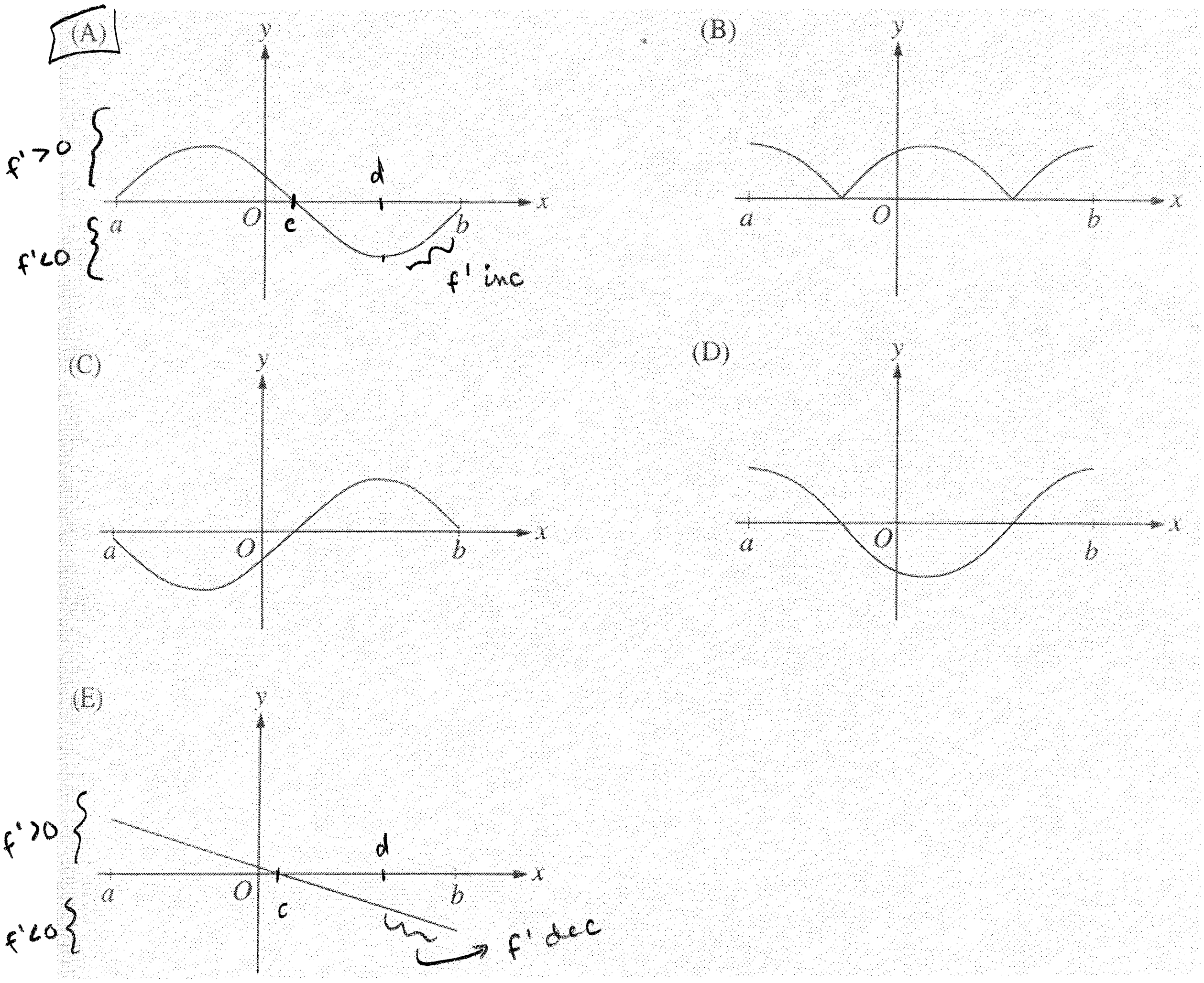
$= \frac{1}{2}(2)^{-1/2} \cdot 2$

$= (2)^{-1/2}$

$= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$



16. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?



17. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) $-2\cos 3$

$\boxed{\text{(B)}}$ $-2\sin 3\cos 3$

(C) $6\cos 3$

(D) $2\sin 3\cos 3$

(E) $6\sin 3\cos 3$

2 chains ... ☺

$f(x) = [\sin(3-x)]^2$

$f'(x) = 2(\sin(3-x)) \cdot \cos(3-x) \cdot -1$

$f'(0) = -2\sin(3-0) \cdot \cos(3-0)$

$= -2\sin 3\cos 3$

18. A differentiable function f has the property that $f(5) = 3$ and $f'(5) = 4$. What is the estimate for $f(4.8)$ using local linear approximation for f at $x = 5$?

$\boxed{\text{(A)}}$ 2.2

(B) 2.8

(C) 3.4

(D) 3.8

(E) 4.6

$y - y_1 = m(x - x_1)$

$y - 3 = 4(x - 5)$

$y - 3 = 4(4.8 - 5)$

$y - 3 = 4(-.2)$

$y = -.8 + 3$

$f(4.8) = 2.2$

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19. The position of a particle moving along a line is given by $x(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $3 < t < 4$ only

(B) $t > 4$ only

(C) $t > 5$ only

(D) $0 < t < 3$ and $t > 5$

$\boxed{\text{(E)}}$ $3 < t < 4$ and $t > 5$

↳ when $a(t)$ + $v(t)$ have same signs (both pos or both neg)

$a(t)$ + $v(t)$ are neg on $(3, 4)$ and

$a(t)$ + $v(t)$ are pos on $(5, \infty)$

$v(t) = 6t^2 - 48t + 90$

$= 6(t^2 - 8t + 15)$

$0 = 6(t-5)(t-3)$

$t = 3, t = 5$

$a(t) = 12t - 48$

$0 = 12(t - 4)$

$t = 4$

$v(t)$: $+$ | $-$ | $+$; 0 1 3 4 5 10

$a(t)$: $-$ | $+$; 0 1 4 10

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20. What is $\lim_{x\to\infty} \frac{x^3 - 9}{3 + 2x - x^2}$?

(A) -3

(B) $-\frac{1}{3}$

(C) -1

(D) 1

$\boxed{\text{(E)}}$ The limit does not exist.

exp N > exp D, so $\lim_{x\to\infty} f(x) = \infty$

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21. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(A) The function has no relative extremum.

(B) The graph of the function has one point of inflection and the function has two relative extrema.

$\boxed{\text{(C)}}$ The graph of the function has two points of inflection and the function has one relative extremum.

(D) The graph of the function has two points of inflection and the function has two relative extrema.

(E) The graph of the function has two points of inflection and the function has three relative extrema.

extrema → $f'(x)$

$f'(x) = 4x^3 - 6x^2$

$0 = 2x^2(2x - 3)$

$x = 0$, $x = 3/2$

f': $-$ | $-$ | $+$; -1 0 1 $3/2$ 2

rel. min @ $x = 3/2$

inf pts → $f''(x)$

$f''(x) = 12x^2 - 12x$

$0 = 12x(x - 1)$

$x = 0$, $x = 1$

f'': $+$ | $-$ | $+$; -1 0 $\frac{1}{2}$ 1 2

inf pts @ $x = 0$, $x = 1$

22. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval [-2, 4] occurs at x =

$f'(x) = 3x^2 - 6x$
$0 = 3x(x-2)$
$x = 0,\ x = 2$

(A) -2 (B) 0 (C) 1 (D) 2 $\boxed{\text{(E)}}$ 4

$f(0) = 12$ $f(-2) = -8$
$f(2) = 8$ $f(4) = 28$

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23. If $f(x) = \sin^{-1} x$, then $f'\left(\frac{1}{2}\right) =$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$f'(\tfrac{1}{2}) = \frac{1}{\sqrt{1-(1/2)^2}} = \frac{1}{\sqrt{1-1/4}} = \frac{1}{\sqrt{3/4}} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$

$\boxed{\text{(A)}}$ $\frac{2\sqrt{3}}{3}$ (B) $\frac{4}{5}$ (C) $-\frac{4}{5}$ (D) $-\frac{2\sqrt{3}}{3}$ (E) $\frac{\pi}{2}$

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24. A spherical balloon is inflated with gas at the rate of 800 cubic cm per minute. How fast is the radius of the balloon increasing at the instant the radius is 30 cm?

(HINT: $V = \frac{4}{3}\pi r^3$)

$\frac{dV}{dt} = 800$ cm^3/min
$r = 30$
$\frac{dr}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$
$800 = 4\pi(30)^2 \frac{dr}{dt}$
$\frac{800}{4\pi(900)} = \frac{dr}{dt}$
$\frac{8}{36\pi} = \frac{dr}{dt}$
$\frac{2}{9\pi}$ cm/min $= \frac{dr}{dt}$

$\boxed{\text{(A)}}$ $\frac{2}{9\pi}$ cm/min (B) $\frac{9}{2\pi}$ cm/min (C) 9 cm/min

(D) 2cm/min (E) None of these

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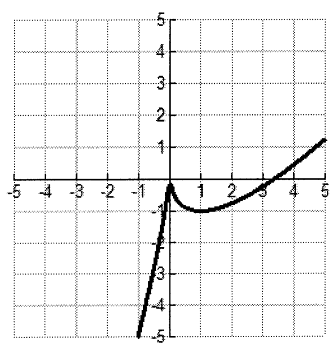
25. The graph shown represents $y = f(x)$. Which of the following is Not True?



(A) f is continuous on the interval [-1,1] → true, no jumps, holes or asymptotes for any x-values

(B) $\lim_{x \to 0} f(x) = f(0)$ → true, no jump, hole, asymptote @ x = 0

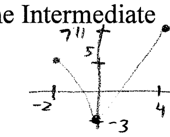
(C) f is concave up on $[0, \infty)$ → true, f(x) curved up on $(0, \infty)$

$\boxed{\text{(D)}}$ f has minimum at (-1,-5) and maximum (1,-1) on the interval [-1,3] false maximum at (0, 1)

(E) All are true

26. If f is continuous on $[-2,4]$ and $f(-2)=5, f(0)=-3,$ *and* $f(4)=711$, then according to the Intermediate Value Theorem, how many zeroes are guaranteed on the closed interval $[-2, 4]$?

(A) none (B) one $\boxed{\text{(C)}}$ two (D) three (E) four



2 zeroes guaranteed

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27. $\lim_{h\to 0}\dfrac{\cos\left(\pi/2+h\right)-\cos\left(\pi/2\right)}{h}=?$

means $f'(x)$ where $f(x)=\cos x$ and value of x is $\pi/2$

$\boxed{\text{(A)}}$ -1
(B) 0
(C) 1
(D) $\cos\left(\pi/2+h\right)$
(E) undefined

$f'(x)=-\sin x$

$f'(\tfrac{\pi}{2})=-\sin\tfrac{\pi}{2}$

$=-1$

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28. Let f and g be differentiable functions with the following properties:

I. $f(x)<0$ for all x.

II. $g(5)=2$

If $h(x)=\frac{f(x)}{g(x)}$ and $h'(x)=\frac{f'(x)}{g(x)}$, then $g(x)=$

(A) $\frac{1}{f'(x)}$
(B) $f(x)$
(C) $-f(x)$
(D) 0
$\boxed{\text{(E)}}$ 2

$h(x)=\dfrac{f(x)}{g(x)} \rightarrow h'(x)=\dfrac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$

$h'(x)=\dfrac{g(x)f'(x)}{(g(x))^2}-\dfrac{f(x)g'(x)}{(g(x))^2}$

$h'(x)=\dfrac{f'(x)}{g(x)}-\dfrac{f(x)g'(x)}{(g(x))^2}$

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since $h'(x)=\dfrac{f'(x)}{g(x)}$, then $\dfrac{f(x)g'(x)}{(g(x))^2}=0$

$\dfrac{f(x)g'(x)}{(g(x))^2}=0$

$g(x)\neq 0$

$f(x)g'(x)=0$

$f(x)=0$ or $g'(x)=0$

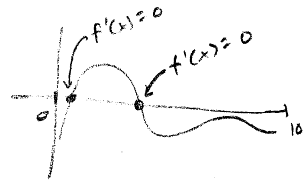
$f(x)<0$ was given

for $g'(x)=0$, then $g(x)$ is a constant

PART II CALCULATOR: MULTIPLE-CHOICE
A calculator MAY be used on this part of the review

1. The first derivative of the function f is given by $f'(x) = \frac{\sin^2 x}{x} - \frac{2}{9}$. How many critical values does f have on the open interval $(0,10)$?

(A) One
(B) Two
(C) Three
(D) Four
(E) Six



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2. If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$. → true, if f diff'able, then f cont.
(B) $\lim_{x\to a} f(x)$ exists. → true continuity @ x=a.
(C) $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists. true def. of derivative @ x=a
(D) $f'(a)$ is defined. → true definition of diff'able @ x=a, derivative @ x=a exists.
(E) $f''(a)$ is defined. 2nd derivative info is unknown.

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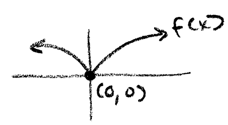
3. Let f be the function given by $f(x) = x^{2/3}$. Which of the following statements about f are true?

I. f is continuous at $x = 0$ true, no jumps holes or asymptotes @ x=0

II. f is differentiable at $x = 0$ → not true $\lim_{x\to 0^-} f'(x) \neq \lim_{x\to 0^+} f'(x)$ "cusp"

III. f has an absolute minimum at $x = 0$ true, min pt (0,0)



(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

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4. If $a \neq 0$, then $\lim_{x\to a} \frac{x^3-a^3}{a^6-x^6}$ is

$$\lim_{x\to a} \frac{x^3-a^3}{(a^3-x^3)(a^3+x^3)} = \lim_{x\to a} \frac{\cancel{x^3-a^3}}{-\cancel{(x^3-a^3)}(a^3+x^3)}$$

$$= \lim_{x\to a} \frac{1}{-(a^3+x^3)}$$

$$= \frac{1}{-(a^3+a^3)}$$

$$= -\frac{1}{2a^3}$$

(A) nonexistent
(B) 0
(C) $-\frac{1}{2a^3}$
(D) $-\frac{1}{a^3}$
(E) $\frac{1}{2a^3}$

5. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$. For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

g dec most rapidly when $g'(x)$ is lowest pt.

(A) 0.949 $\boxed{\text{(B)}}$ 2.017 (C) 3.106 (D) 5.965 (E) 8.000

$g'(x)$ $(2.017, -35.962)$

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6. If the length l of a rectangle is decreasing at a rate of 2 inches per minutes while its width w is increasing at a rate of 2 inches per minute, which of the following must be true about the area A of the rectangle?

(A) A is always increasing.

(B) A is always decreasing.

$\boxed{\text{(C)}}$ A is increasing only when $l > w$.

(D) A is increasing only when $l < w$.

(E) A remains constant.

$A = lw$

$\frac{dA}{dt} = w\frac{dl}{dt} + l\frac{dw}{dt}$

$\frac{dA}{dt} = w(-2) + l(2)$

$\frac{dA}{dt} = -2w + 2l > 0$

$2l > 2w$

$l > w$

$\frac{dl}{dt} = -2$ in/min

$\frac{dw}{dt} = 2$ in/min

$\frac{dA}{dt} = ?$

$\frac{dA}{dt} > 0$ when $l > w$, so A inc when $l > w$

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7. Which of the following is an equation of the line tangent to the graph of $f(x) = x^6 - x^4$ at the point where $f'(x) = -1$?

$\boxed{\text{(A)}}$ $y = -x - 1.031$

(B) $y = -x - 0.836$

(C) $y = -x + 0.836$

(D) $y = -x + 0.934$

(E) $y = -x + 1.031$

$f'(x) = 6x^5 - 4x^3$

$6x^5 - 4x^3 = -1$

$x = -.934$

$f(-.934) = -.098$

$y - -.098 = -1(x - -.934)$

$y + .098 = -x - .934$

$y = -x - 1.031$

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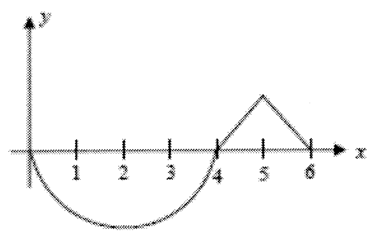
8. Let f be the function given by $f(x) = \tan x$ and let g be the function given by $g(x) = x^3$. At what value of x in the interval $0 \le x \le \pi$ do the graphs of f and g have parallel tangent lines? $\to$ $f'(x) = g'(x)$

(A) 0 (B) 0.75 $\boxed{\text{(C)}}$ 1.883 (D) 1.697 (E) 10.63



$f(x) = g'(x)$

g': $-$ on $(0,4)$, $+$ on $(4,6)$; 0, 4, 6

$g' > 0$ means g inc on $(4, 6)$

9. The graph of f given above consists of two line segments and a semicircle. If $f(x) = g'(x)$, for what values of x is g increasing?

(A) (0,4) (B) (2,4) only (C) (2,5) (D) (4,5) only $\boxed{\text{(E)}}$ (4,6)

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10. The graph of the function $y = x^5 - x^2 + \sin x$ has a point of inflection at $x =$ $\to$ when y'' changes signs

(A) 0.324 $\boxed{\text{(B)}}$ 0.499 (C) 0.506 (D) 0.611 (E) 0.704

$y'' = 0$ @ $x = 0.499$

---

11. Let h be the function defined by $h(x) = \cos 3x + \ln 4x$. What is the least value of x at which the graph of h changes concavity? $\to$ when $h''(x)$ changes signs

(A) 1.555 $\boxed{\text{(B)}}$ 0.621 (C) 0.371 (D) 0.096 (E) 0.004

12. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - x - 12)g(x)$, which of the following is true?

$f'(x) = (x-4)(x+3)\,g(x)$

$0 = (x-4)(x+3)\,g(x)$ → remember $g(x) < 0$

$x = 4,\ x = -3$

(A) f has a relative maximum at $x = -3$ and a relative minimum at $x = 4$

$\boxed{\text{(B)}}$ f has a relative ~~maximum~~ minimum at $x = -3$ and a relative ~~minimum~~ maximum at $x = 4$

(C) f has a relative maximum at $x = +3$ and a relative minimum at $x = -4$

(D) f has a relative ~~maximum~~ minimum at $x = +3$ and a relative ~~minimum~~ maximum at $x = 4$

(E) ~~f has a relative maximum at $x = -3$ and a relative minimum at $x = 4$~~
No relative extrema.

f': $-$ $+$ $-$

-3 ☺ 4 ☹

rel. min @ $x = -3$

rel. max @ $x = 4$

13. A particle moves on the x-axis with velocity given by $v(t) = 3t^4 - 11t^2 + 9t - 2$ for $-3 \le t \le 3$. How many times does the particle change direction as t increases from –3 to 3?

↳ when velocity changes signs

(A) zero (B) one $\boxed{\text{(C)}}$ two (D) three (E) four

$v(t)$; -3; 3; $v(t)$ changes signs twice

14. Let f be the function with the derivative given by $f'(x) = \cos(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

↳ when f' changes signs

(A) One (B) Two $\boxed{\text{(C)}}$ Three (D) Four (E) Five

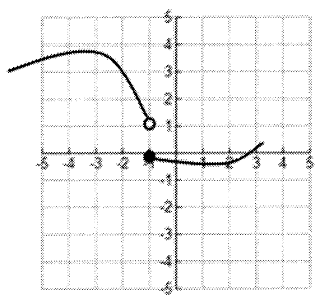
$f'(x)$

15. The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity? → highest velocity

$v(t)$ max @ $t = .316$

$h(.316) = 10.263$

(A) 2.545 meters $\boxed{\text{(B)}}$ 10.263 meters (C) 34.125 meters (D) 54.889 meters (E) 89.005 meters



16. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $f(-1) = 1$

(B) $\lim_{x\to -1} f(x) = 0$

(C) $\lim_{x\to -1} f(x) = 1$

$\boxed{\text{(D)}}$ $\lim_{x\to -1} f(x)$ does not exist

(E) $f(-1)$ does not exist

$\lim_{x\to -1^-} f(x) = 1 \neq \lim_{x\to -1^+} f(x) = 0$

17. The function f has first derivative given by $f'(x) = \frac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f?

f'' changes signs

(A) 1.008 $\boxed{\text{(B)}}$ 0.473 (C) 0

(D) –0.278 (E) The graph of f has no inflection points

@ $x = .473$

$f''(x)$

PART III CALCULATOR: FREE-RESPONSE

A calculator MAY be used on this part of the review.

1. The function f is continuous on the closed interval [0, 10] and has values that are given in the table below. Using five equal subintervals, what is the left sum, right sum, midpoint sum, and trapezoidal approximations of $\int_0^{10} f(x)\,dx$?

$\Delta x = \frac{10-0}{5} = 2$

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	19.5	18	15.5	12	7.5	2	-4.5	-12	-20.5	-30

left sum $\approx 2(20+18+12+2+-12) = 80$ right sum $\approx 2(-30+-12+2+12+18) = -20$

midpt sum $\approx 2(19.5+15.5+7.5+-4.5+-20.5) = 35$

2. For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$. Show that the number of mosquitoes is increasing at time $t = 6$. At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate?

$R(6) = 5\sqrt{6}\cos(6/5) = 4.438 > 0$, $R(6) > 0$, So # mosquitoes increasing @ $t=6$

$R'(6) = -1.913 < 0$, $\therefore$ # mosquitoes is increasing @ a decreasing rate @ $t=6$

PART IV NON-CALCULATOR: FREE-RESPONSE

NO calculator may be used on this part of the review.

1. The function $f(x) = x^3 + ax^2 + bx + c$ has a relative maximum at $(-3, 25)$ and a point of inflection at $x = -1$. Find a, b, and c.

↳ $f''=0$ or DNE and f'' changes signs

$f'=0$ or DNE and $f'(x)$ changes from pos to neg.

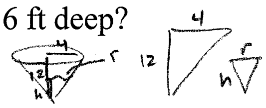
2. Water is being pumped into a conical reservoir (vertex down) at the constant rate of 10π ft^3/min. If the reservoir has a radius of 4 ft and is 12 ft deep, how fast is the water rising when the water is 6 ft deep?

$\frac{dh}{dt} = \frac{5}{2}$ ft/min

$\frac{dV}{dt} = 10\pi$ ft^3/min

$h = 6$ ft

$\frac{dh}{dt} = ?$



$\frac{4}{r} = \frac{12}{h}$

$4h = 12r$

$\frac{1}{3}h = r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$10\pi = \frac{1}{9}\pi(6)^2 \frac{dh}{dt}$

$10\pi = 4\pi \frac{dh}{dt}$

3. State the set of values for which $f(x) = (x-2)(x-3)^2$ is BOTH increasing and concave up.

$f'>0$ $f''>0$

① $f'(x) = 3x^2 + 2ax + b$

$f'(-3) = 3(-3)^2 + 2a(-3) + b$

$0 = 27 - 6a + b$

$0 = 27 - 6(3) + b$

$0 = 27 - 18 + b$

$0 = 9 + b$

$\boxed{-9 = b}$

$f''(x) = 6x + 2a$

$f''(-1) = 6(-1) + 2a$

$0 = -6 + 2a$

$6 = 2a$

$\boxed{3 = a}$

$f(-3) = (-3)^3 + 3(-3)^2 + -9(-3) + c$

$25 = -27 + 27 + 27 + c$

$25 = 27 + c$

$\boxed{-2 = c}$

③ $f'(x) = (x-3)^2(1) + (x-2)[2(x-3)(1)]$

$= (x-3)^2 + 2(x-2)(x-3)$

$= (x-3)[x-3+2(x-2)]$

$0 = (x-3)(3x-7)$

$x = 3,\ x = 7/3$

f': $+$ | $-$ | $+$; 0 $\ 7/3\ $ 2.5 $\ 3\ $ 5

$f'(x) = 3x^2 - 16x + 21$

$f''(x) = 6x - 16$

$0 = 2(3x - 8)$

$x = 8/3$

f'': $-$ | $+$; 0 $\ 8/3\ $ 5

f inc + concave up on $(3, \infty)$