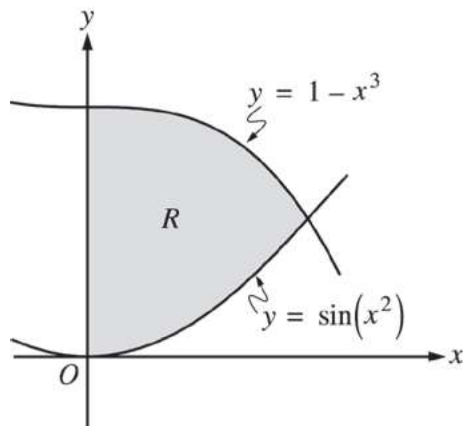

1. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$, as shown in the figure above.

(a) Find the area of R.

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Continue problem 1 on page 5.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

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2. The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years, for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$$B(t) = 1000e^{0.06t} \text{ penguins per year}$$

and the death rate for the penguins on the island is modeled by

$$D(t) = 250e^{0.1t} \text{ penguins per year.}$$

(a) What is the rate of change of the penguin population on the island at time $t = 0$?

(b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?

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Continue problem 2 on page 7.

(c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \le t \le 40$?

(d) To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \le t \le 40$. Show the analysis that leads to your answers.

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NO CALCULATOR ALLOWED

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

3. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

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(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

Continue problem 3 on page 15.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

4. Let f be the function given by $f(x) = (x^2 - 2x - 1)e^x$.

(a) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

$\lim_{x\to\infty} f(x) =$ ____________

$\lim_{x\to-\infty} f(x) =$ ____________

(b) Find the intervals on which f is increasing. Show the analysis that leads to your answer.

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Continue problem 4 on page 17.

(c) Find the intervals on which the graph of f is concave down. Show the analysis that leads to your answer.

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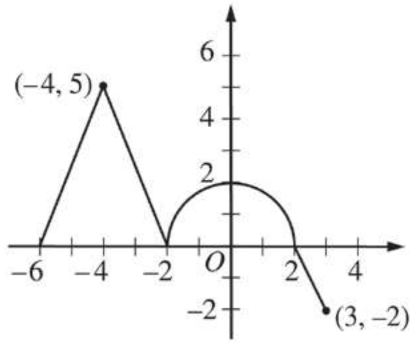

Graph of f

5. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

Continue problem 5 on page 19.

(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

NO CALCULATOR ALLOWED

6. Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f, the slope is given by $\frac{3x^2}{y}$.

   (a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

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Continue problem 6 on page 21.

NO CALCULATOR ALLOWED

(b) Find an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = \frac{3x^2}{y}$ with the initial condition $f(2) = -8$.

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