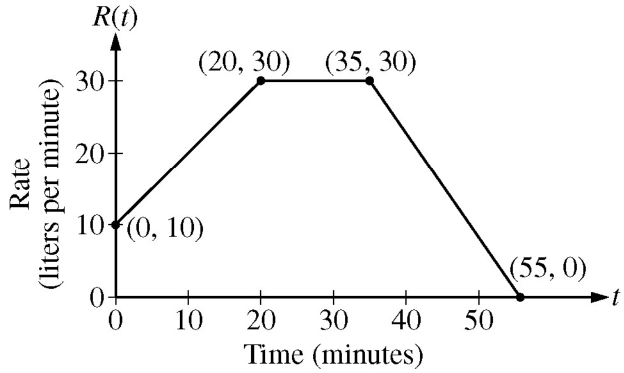


1. At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.

   (a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

   (b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

Do not write beyond this border.

Continue problem 1 on page 5.

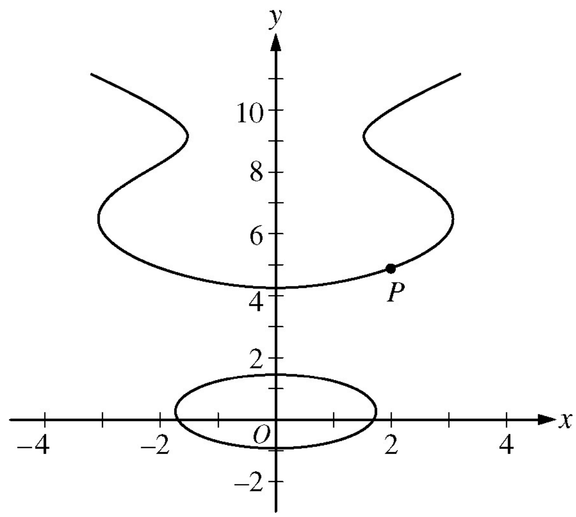
(c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

(d) Using the functions R and D, determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.



2. The graph of the equation $x^2 = -2 + y + 5\cos y$ is shown above for $y \leq 11$. It is known that $\dfrac{dy}{dx} = \dfrac{2x}{1 - 5\sin y}$.

The x-coordinate of point P shown on the graph is 2.

(a) Write an equation for the line tangent to the graph at point P.

Do not write beyond this border.

Continue problem 2 on page 7.

(b) For $y \leq 11$, find the y-coordinate of each point on the graph where the line tangent to the graph at that point is vertical.

(c) Find the average value of the x-coordinates of the points on the graph in the first quadrant between $y = 5$ and $y = 9$.

Do not write beyond this border.

NO CALCULATOR ALLOWED

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

3. Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \leq 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

(a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

Do not write beyond this border.

Do not write beyond this border.

Continue problem 3 on page 15.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \dfrac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.

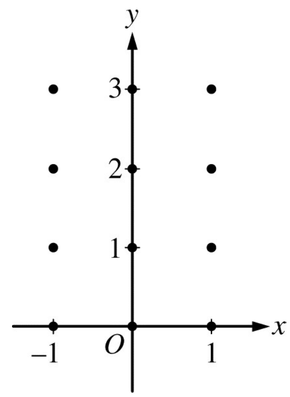
Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

4. Consider the differential equation $\frac{dy}{dx} = \frac{x(y-1)}{4}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

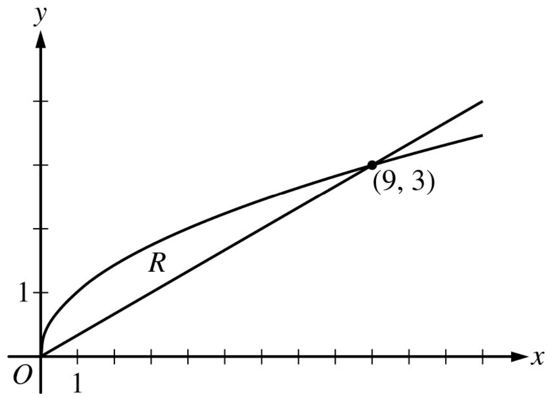
Do not write beyond this border.

Do not write beyond this border.

Continue problem 4 on page 17.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 3$.

Do not write beyond this border.



5. Let R be the region in the first quadrant enclosed by the graphs of $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{3}$, as shown in the figure above.

(a) Find the area of region R.

Continue problem 5 on page 19.

NO CALCULATOR ALLOWED

(b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when R is revolved about the horizontal line $y = 4$.

(c) Find the maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$. Justify your answer.

NO CALCULATOR ALLOWED

6. Let $g(x) = 4(x+1)^{-2/3}$ and let f be the function defined by $f(x) = \int_0^x g(t)\,dt$ for $x \geq 0$.

(a) Find $f(26)$.

(b) Determine the concavity of the graph of $y = f(x)$ for $x > 0$. Justify your answer.

Do not write beyond this border.

Continue problem 6 on page 21.

(c) Let h be the function defined by $h(x) = x - f(x)$. Find the minimum value of h on the interval $0 \le x \le 26$.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.