t (minutes)	0	2	5	7	10
$h(t)$ (inches)	3.5	10.0	15.5	18.5	20.0

1. The depth of water in tank A, in inches, is modeled by a differentiable and increasing function h for $0 \le t \le 10$, where t is measured in minutes. Values of $h(t)$ for selected values of t are given in the table above.

(a) Use the data in the table to find an approximation for $h'(6)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the value of $\int_0^{10} h(t)\, dt$ using a right Riemann sum with the four subintervals indicated by the data in the table. Is this approximation greater than or less than $\int_0^{10} h(t)\, dt$? Give a reason for your answer.

(c) The depth of water in tank B, in inches, is modeled by the function $g(t) = 3.2 + 17.5\sqrt{\sin(0.16t)}$ for $0 \le t \le 10$, where t is measured in minutes. Find the average depth of the water in tank B over the interval $0 \le t \le 10$. Is this value greater than or less than the average depth of the water in tank A over the interval $0 \le t \le 10$? Give a reason for your answer.

(d) According to the model given in part (c), is the depth of the water in tank B increasing or decreasing at time $t = 6$? Give a reason for your answer.

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2. Particle Q moves along the x-axis so that its velocity at any time t is given by $v_Q(t) = 1 - 3\cos\left(\frac{t^2}{5}\right)$, and its acceleration at any time t is given by $a_Q(t) = \frac{6t}{5}\sin\left(\frac{t^2}{5}\right)$. The particle is at position $x = 2$ at time $t = 0$.

(a) In the interval $0 < t < 5$, when is the velocity of particle Q increasing? Give a reason for your answer.

(b) Find the position of particle Q at time $t = 3$.

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(c) A second particle, R, moves along the x-axis so that its position at any time t is given by a differentiable function $x_R(t)$, where $x_R(1) = 4$ and $x_R(3) = 8$. Explain why there must be a time t, for $1 < t < 3$, at which the velocity of particle R is 2.

(d) At time $t = 3$, the velocity of particle R described in part (c) is -2. Are particles Q and R moving toward each other or away from each other at time $t = 3$? Explain your reasoning.

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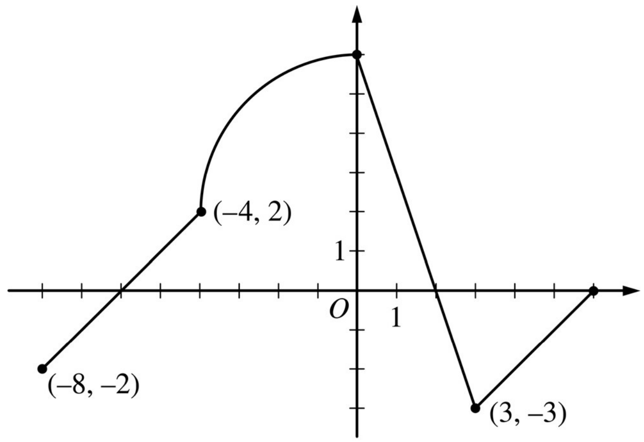

Graph of g

3. A continuous function g is defined on the closed interval $-8 \leq x \leq 6$. The graph of g, shown above, consists of three line segments and a quarter of a circle centered at the point $(0, 2)$. Let f be the function given by $f(x) = \int_{-8}^{x} g(t)\, dt$.

   (a) Find all values of x in the interval $-8 < x < 6$ at which f has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

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(b) Find $f(0)$.

(c) Find $\lim_{x \to -4} \frac{f(x)}{x^2 + 4x}$.

(d) Let h be the function defined by $h(x) = \frac{g(x)}{x^2 + 1}$. Find $h'(1)$.

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4. Consider the differential equation $\frac{dy}{dx} = (y - 2)(x^2 + 1)$.

(a) Find $y = g(x)$, the particular solution to the given differential equation with initial condition $g(0) = 5$.

(b) For the particular solution $y = g(x)$ found in part (a), find $\lim_{x \to -\infty} g(x)$.

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(c) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 3$. Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 3)$. Is the graph of $y = f(x)$ concave up or concave down at the point $(1, 3)$? Give a reason for your answer.

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5. The function f is defined by

$$f(x) = \begin{cases} 3x^2 + 2x & \text{for } x \le 0 \\ e^{2x} + 2 & \text{for } x > 0. \end{cases}$$

(a) Is f continuous at $x = 0$? Justify your answer.

(b) Find $f'(-2)$ and $f'(3)$.

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(c) Explain why $f'(0)$ does not exist.

(d) Let g be the function given by $g(x) = \int_{-1}^{x} f(t)\, dt$. Find $g(1)$.

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6. A hive contains 35 hundred bees at time $t = 0$. During the time interval $0 \leq t \leq 4$ hours, bees enter the hive at a rate modeled by $E(t) = 16t - 3t^2$, where $E(t)$ is measured in hundreds of bees per hour. During the same time interval, bees leave the hive at a rate modeled by $L(t) = -2t + 15$, where $L(t)$ is measured in hundreds of bees per hour.

(a) How many bees leave the hive during the time interval $0 \leq t \leq 2$?

(b) Write an expression involving one or more integrals for the total number of bees, in hundreds, in the hive at time t for $0 \leq t \leq 4$. Find the total number of bees in the hive at $t = 4$.

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(c) Find the minimum number of bees in the hive for $0 \leq t \leq 4$. Justify your answer.

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