CollegeBoard
Advanced Placement
Program

AP® Calculus AB
2010 Free-Response Questions
Form B

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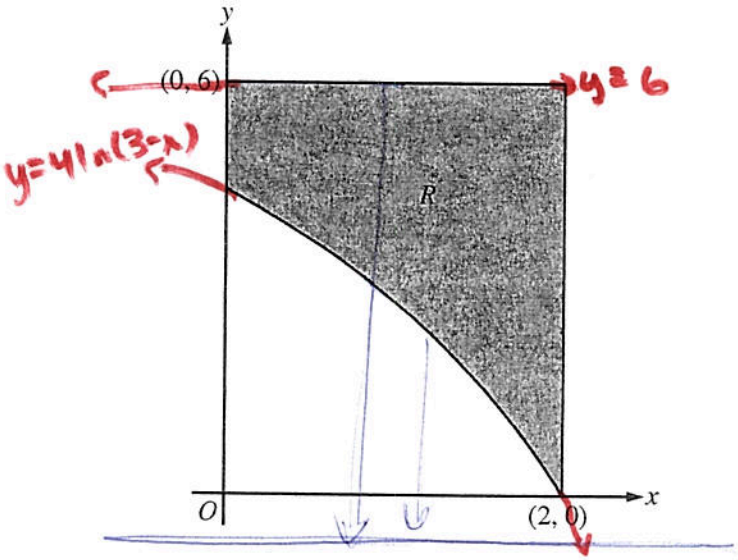
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R.

Area of R = $\int_0^2 (6 - 4\ln(3-x))\,dx$

1pt - integrand

= ~~8.112~~

= 6.817

1pt - answer

1pt - correct limits (a), (b) or (c)

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Continue problem 1 on page 5.

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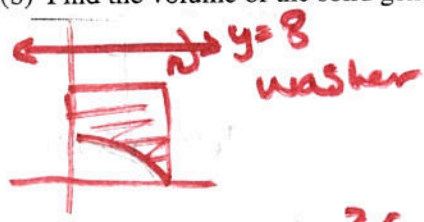
(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.



outside radius inside radius

$$V = \pi \int_0^2 \left[\left(8 - 2\ln(3-x)\right)^2 - (8-6)^2 \right] dx$$

2pts - integrand

1pt - answer

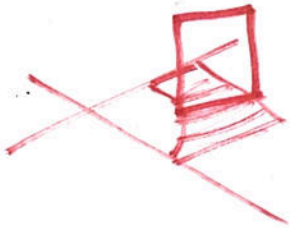
$= 168.180$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

x's

$A = (\text{side})^2$

$$V = \int_0^2 \left(6 - 4\ln(3-x)\right)^2 dx$$



$= 26.267$

$A = \left(6 - 4\ln(3-x)\right)^2$

2pts - integrand

1pt - answer

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2. The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

(a) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

$\hookrightarrow g' = 0$

$g'(x) = 0$

$x = .163$, $x = .359$

1pt - $g'(x) = 0$

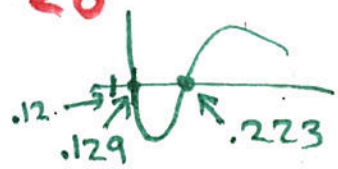
g has horizontal tangent lines @ $x = .163$ and $x = .359$

1pt - answer

(b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

$\hookrightarrow g'' < 0$

$g'' < 0$ on $(.129, .223)$



g concave down on $(.129, .223)$ b/c $g'' < 0$ on that interval.

1pt - answer
1pt - reason

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Continue problem 2 on page 7.

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(c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

$y - y_1 = m(x - x_1)$

1pt - $g'(.3)$

1pt - equation

$y - 1.546 = -.472(x - .3)$

$g'(.3) = -.472$

1pt - integral

$g(.3) = 2 + \int_1^{.3} g'(x)\,dx$

$= 1.546$ 1pt - $g(.3)$

$g(t) = 2 + \int_1^{t} g'(x)\,dx$

$g(.3) = 2 - \int_{.3}^{1} g'(x)\,dx$

(d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

check concavity

on $(.3, 1)$, $g'' > 0$

Tangent line to g @ $x = .3$ lies below graph of g b/c $g'' > 0$ (g concave up)

1pt - answer w/reason

up

down

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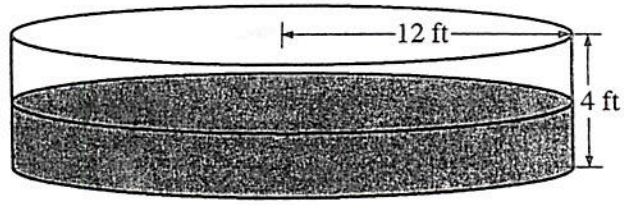
3 3 3 3 3 3 3 3 3

midpt midpt midpt

hr

ft³/hr

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

total amount of water pumped in $= 4(46) + 4(57) + 4(62)$

$= 660 \text{ ft}^3$

$(\text{ft}^3/\text{hr})(\text{hr})$

1pt - midpt sum

1pt - answer

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

Amount water leaked out $= \int_0^{12} R(t)\,dt$

$= 225.594 \text{ ft}^3$

1pt - integral

1pt - answer

ft^3/hr

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Continue problem 3 on page 9.

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(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

volume of Water in pool = initial water in pool + water pumped in − water leaked out

$V(12) = 1000 + 660 - \int_0^{12} R(t)\,dt$

$= 1434.406$

$= 1434 \text{ ft}^3$

1 pt – answer

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

$\frac{dh}{dt} = ?$

(volume of Water in pool)′ = rate pumped in − rate ~~pumped~~ leaked out

$V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8)$

$= 43.242 \text{ ft}^3/\text{hr}$

1 pt – $V'(8)$

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

$43.242 = \pi (12)^2 \frac{dh}{dt}$

$.096 = \frac{dh}{dt}$

1 pt – $\left.\frac{dh}{dt}\right|_{t=8}$

1 pt → $\frac{dV}{dt}$ equation

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

water level rising @ $t = 8$ at $.096$ ft/hr

1 pt – units for $V'(8)$ and $\frac{dV}{dt}$

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