CollegeBoard
Advanced Placement
Program

# AP® Calculus AB
# 2011 Free-Response Questions

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## CALCULUS AB
## SECTION II, Part A
**Time—30 minutes**
**Number of problems—2**

**A graphing calculator is required for these problems.**

1. For $0 \le t \le 6$, a particle is moving along the $x$-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

   (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$ ? Give a reason for your answer.

   (b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

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Continue problem 1 on page 5.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

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| $t$ (minutes) | 0 | 2 | 5 | 9 | 10 |
|---|---|---|---|---|---|
| $H(t)$ (degrees Celsius) | 66 | 60 | 52 | 44 | 43 |

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function $H$ for $0 \le t \le 10$, where time $t$ is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time $t$ are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

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Continue problem 2 on page 7.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time $t$ is modeled by a differentiable function $B$ for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

**END OF PART A OF SECTION II**

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.**

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**NO CALCULATOR ALLOWED**

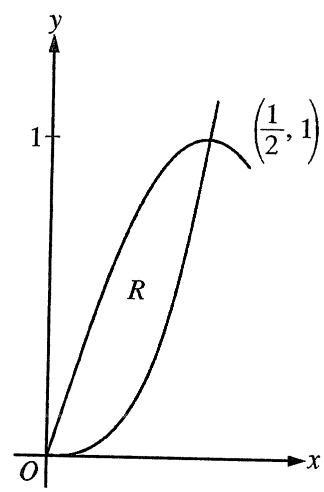
**CALCULUS AB**

**SECTION II, Part B**

**Time—60 minutes**

**Number of problems—4**

**No calculator is allowed for these problems.**




3. Let $R$ be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

   (a) Write an equation for the line tangent to the graph of $f$ at $x = \frac{1}{2}$.

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Continue problem 3 on page 9.

(b) Find the area of $R$.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when $R$ is rotated about the horizontal line $y = 1$.

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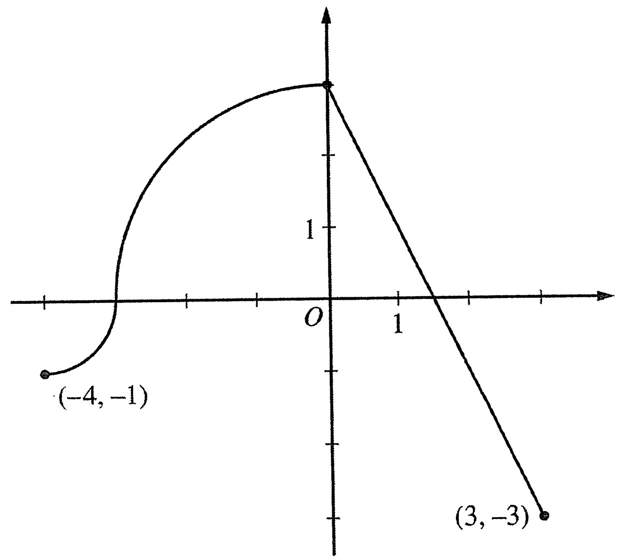
**NO CALCULATOR ALLOWED**




Graph of $f$

4. The continuous function $f$ is defined on the interval $-4 \le x \le 3$. The graph of $f$ consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the $x$-coordinate of the point at which $g$ has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

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Continue problem 4 on page

(c) Find all values of $x$ on the interval $-4 < x < 3$ for which the graph of $g$ has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of $f$ on the interval $-4 \le x \le 3$. There is no point $c$, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

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**NO CALCULATOR ALLOWED**

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function $W$ models the total amount of solid waste stored at the landfill. Planners estimate that $W$ will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. $W$ is measured in tons, and $t$ is measured in years from the start of 2010.

(a) Use the line tangent to the graph of $W$ at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

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(b) Find $\frac{d^2W}{dt^2}$ in terms of $W$. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

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Continue problem 5 on page 13.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

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**NO CALCULATOR ALLOWED**

6. Let $f$ be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that $f$ is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of $x$ for which $f'(x) = -3$.

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Continue problem 6 on page 15.

(c) Find the average value of $f$ on the interval $[-1, 1]$.

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