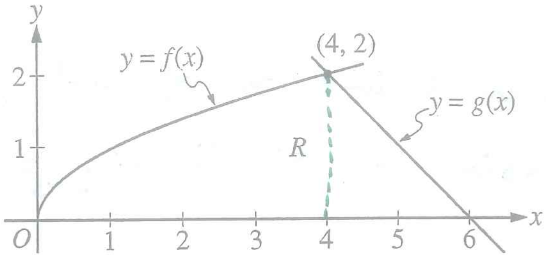
NO CALCULATOR ALLOWED

CALCULUS AB
SECTION II, Part B

Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



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3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

Area of R = $\int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx$

$= \frac{2}{3}x^{3/2}\Big|_0^4 + (6x - \frac{1}{2}x^2)\Big|_4^6$

$= \frac{2}{3}(4)^{3/2} + (6(6) - \frac{1}{2}(6)^2 - (6\cdot 4 - \frac{1}{2}(4)^2)$ ← ok to stop here.

$= \frac{16}{3} + 36 - 18 - 24 + 8$

$= \frac{16}{3} + 2 = \boxed{\frac{22}{3}}$

1pt-integral
1pt-antiderive
1pt-answer

OR

Area of R = $\int_0^2 (6 - y - y^2)\,dy$

$= (6y - \frac{1}{2}y^2 - \frac{1}{3}y^3)\Big|_0^2$

$= 12 - 2 - \frac{8}{3}$

$= 22/3$

$y = 6 - x$
$y + x = 6$
$x = 6 - y$

$y = \sqrt{x}$
$y^2 = x$

Continue problem 3 on page 9.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

work in y's

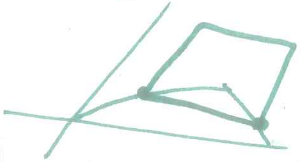
work in y's

$y = 6 - x$ $\quad y = \sqrt{x}$

$x = 6 - y$ $\quad y^2 = x$

$A = bh$

$A = (6-y-y^2)(2y)$



base $= (6-y-y^2)$

Volume $= \int_0^2 (6-y-y^2)(2y)\ dy$

2pts - integrand

1pt - answer

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

→ (x, y)

$f'(x)$ =

↳ need slope of g and then opp. reciprocal.

$g'(x) = -1$

m ⊥ to g is 1

~~PSF~~

~~II~~

$f'(x) = \frac{1}{2}x^{-1/2}$

$1 = \frac{1}{2}x^{-1/2}$

$2 = x^{-1/2} \rightarrow \frac{2}{1} = \frac{1}{x^{1/2}}$

$\frac{1}{2} = x^{1/2}$

$\frac{1}{4} = x$

Point P is $(\frac{1}{4}, \frac{1}{2})$

$f(\frac{1}{4}) = \sqrt{\frac{1}{4}} = \frac{1}{2}$

1pt - $f'(x)$

1pt - equation

1pt - answer

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4. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

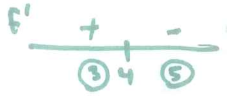
$\rightarrow f' = 0$ OR DNE

(a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

$f'(x) = 0$ or DNE

crit pt of f @ $x = 4$, ~~$x = 0$~~

↳ not in domain



f rel. max @ $x = 4$ b/c f' changes from pos to neg @ $x = 4$.

1pt – $x = 4$

1pt – rel. max

1pt – reason

(b) Find all intervals on which the graph of f is concave down. Justify your answer.

$f'' < 0$

$$f''(x) = x^{-3}(-1) + (4-x)(-3x^{-4})$$

$$= -x^{-3} - 3x^{-4}(4-x)$$

$$= -x^{-3} - 12x^{-4} + 3x^{-3}$$

$$= -12x^{-4} + 2x^{-3} = -\frac{12}{x^4} + \frac{2}{x^3} = \frac{-12 + 2x}{x^4}$$

$$= x^{-4}(-12 + 2x)$$

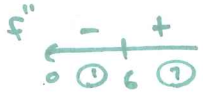
2pts – $f''(x)$

$-12 + 2x = 0$

$2x = 12$

$x = 6$

~~$x = 0$~~ not in domain



f concave down on $(0, 6)$ b/c $f'' < 0$ on that interval.

1pt – answer w/ reason

Continue problem 4 on page 11.

$f(3) = 2+\int_1^3 f'(t)dt$

(c) Given that $f(1) = 2$, determine the function f.

$f(x) = 2 + \int_1^x (4-t)t^{-3}\,dt$ 1pt - integral

$= 2 + \int_1^x (4t^{-3} - t^{-2})\,dt$

$= 2 + \left[4(\frac{1}{-2}t^{-2}) - \frac{1}{-1}t^{-1}\right]\Big|_1^x$ 1pt- antiderivative

$= 2 + (-2t^{-2} + t^{-1})\Big|_1^x$

$= 2 + -2x^{-2} + x^{-1} - (-2(1)^{-2} + 1^{-1})$ ← ok to stop here

$= 2 - 2x^{-2} + x^{-1} - (-1)$ 1pt - answer

$f(x) = 3 - 2x^{-2} + x^{-1}$

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GO ON TO THE NEXT PAGE.

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

5. Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

↳ $v'(t)$

$a(5) = \frac{v(10)-v(0)}{10-0}$ $\frac{m/sec}{sec}$ … ☺

$= \frac{2.3-2.0}{10}$

1 pt - answer

$= \frac{.3}{10}$ m/sec² OR .03 m/sec²

1 pt - units in (a) or (b)

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

$\int_0^{60} |v(t)|\, dt = 10(2) + 30(2.3) + 20(2.5)$

$= 20 + 69 + 50$

$= 139$ meters

1pt - left approx.

$\int_0^{60} |v(t)|\, dt$ is the total distance Ben traveled on his unicycle in meters from $t=0$ to $t=60$ sec.

1pt - meaning

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Continue problem 5 on page 13.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

IVT, MVT

MVT $B'(t) = \frac{B(b)-B(a)}{b-a}$

B(t) is cont + diff'able b/c B is twice-diff'able

$v(t)$ or $B'(t) = \frac{B(60)-B(40)}{60-40}$

$= \frac{49-9}{20}$

$= 2$

1pt - difference quotient

Since $B'(t)=2$ on $(40,60)$, there must be a time t when $v(t)=2$ on $(40,60)$

1pt - conclusion w/reason

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

$\hookrightarrow L'(40) = ?$

$[L(t)]^2 = 12^2 + (B(t))^2$

$2(L(t)) \cdot L'(t) = 2B(t) \cdot B'(t)$

$2L(40) \cdot L'(40) = 2B(40) \cdot B'(40)$

$2(15) \cdot L'(40) = 2(9) \cdot 2.5$

$L'(40) = \frac{\cancel{2} \cdot 9 \cdot 2.5}{\cancel{2} \cdot 15}$

$L'(40) = \frac{3(\frac{5}{2})}{5} = 3(\frac{5}{2}) \div 5 = \frac{3}{1}(\frac{5}{2}) \cdot \frac{1}{5}$

$= \frac{3}{2}$ m/sec

1pt - derivatives
1pt - use $B'(t) = v(t)$

$(L(40))^2 = 12^2 + (B(40))^2$

$[L(40)]^2 = 12^2 + 9^2$

$L(40) = \sqrt{144+81}$

$= \sqrt{225}$

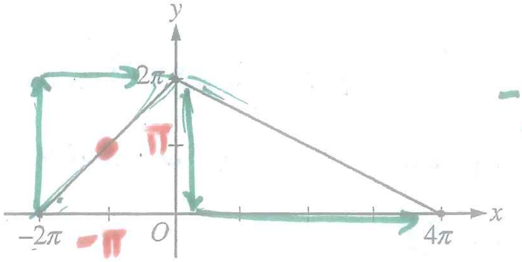
$= 15$

18
4.5
22.5

1pt - answer

$\frac{2\pi}{2\pi} = 1$

$\frac{-2\pi}{4\pi} = -\frac{1}{2}$



Graph of g

6. Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

$$\int_{-2\pi}^{4\pi} \left(g(x) - \cos\left(\frac{x}{2}\right)\right) dx$$

$$= \int_{-2\pi}^{4\pi} g(x)\,dx - \int_{-2\pi}^{4\pi} \cos\left(\frac{x}{2}\right) dx$$

$$= \frac{1}{2}(6\pi)(2\pi) - \int_{-\pi}^{2\pi} \cos \cdot u \cdot 2du$$

$$= 6\pi^2 - 2\sin u \Big|_{-\pi}^{2\pi}$$

$$= 6\pi^2 - 2(\sin 2\pi - \sin(-\pi))$$ ← STOP HERE

$$= 6\pi^2 - 2(0 - 0)$$

$$= 6\pi^2$$

$u = \frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}$

$2du = dx$

$u(-2\pi) = \frac{-2\pi}{2} = -\pi$

$u(4\pi) = \frac{4\pi}{2} = 2\pi$

1pt - antiderive

1pt - answer

Continue problem 6 on page 15.