AP®

AP® Calculus AB
2012 Free-Response Questions

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t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) = \frac{W(15) - W(9)}{15 - 9}$$

°F/min ... ☺

$$= \frac{67.9 - 61.8}{6}$$

$$= 1.017 \text{ °F/min}$$

1pt - estimate

$W'(12)$ means that water temperature increasing at a rate of 1.017 °F/min @ $t = 12$ min.

OR rate of change of water temp is 1.017 °F/m @ $t = 12$ min

1pt - meaning w/ units

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)\,dt = W(t)\Big|_0^{20}$$

$$= W(20) - W(0)$$

← w measured in °F ... ☺

$$= 71 - 55$$

$$= 16 \text{ °F}$$

1pt - value

$\int_0^{20} W'(t)\,dt$ means water temperature changed 16°F from $t=0$ to $t = 20$ minutes

1pt - meaning w/ units

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Continue problem 1 on page 5.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\frac{1}{20}\int_0^{20} W(t)\,dt = \frac{1}{20}\left[55(4) + 57.1(5) + 61.8(6) + 67.9(5)\right]$$

$$= 60.79 \ ^\circ F$$

1pt - left sum

1pt - approx

W(t) is increasing, so left Reimann sum is an underestimate

1pt - underest. w/ reason

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(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

initial

↳ $\int W'(t)$

$$W(25) = \underbrace{71}_{W(20)} + \int_{20}^{25} W'(t)\,dt$$

1pt - integral

$$= 73.043$$

1pt - answer

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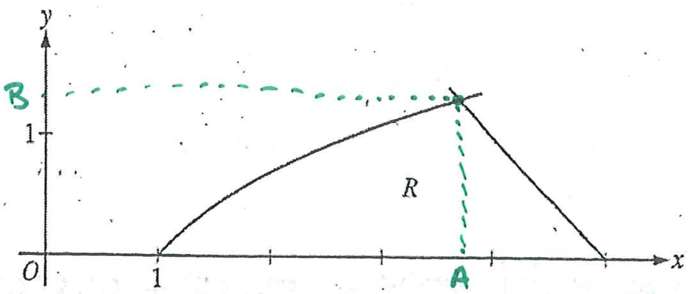

2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

$(A, B) = (3.693, 1.307)$

Area of $R = \int_1^A \ln x \, dx + \int_A^5 (5-x)\, dx$

$= 2.986$

1pt - integrand

1pt - limits

1pt - answer

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Continue problem 2 on page 7.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

work in x's

$A = (\text{side})^2$

$$V = \int_1^A (\ln x)^2\,dx + \int_A^5 (5-x)^2\,dx$$

* two different squares!!

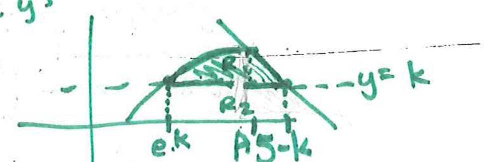
$A = (\ln x)^2$ $A = (5-x)^2$

2pts - integrands

1pt - total volume expression

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

easier to work in y's



$$\frac{\text{Area } R}{2} = \int_0^k (5 - y - e^y)\,dy$$

$y = 5-x$ $y = \ln x$

$5 - y = x$ $e^y = x$

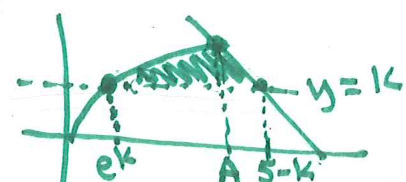
$$\frac{2.986}{2} = \int_0^k (5 - y - e^y)\,dy$$

$k = \ln x$ $k = 5-x$

$e^k = x$ $5-k = x$

OR $$\frac{2.986}{2} = \int_k^B (5 - y - e^y)\,dy$$

OR $$\frac{2.986}{2} = \int_{e^k}^{A} (\ln x - k)\,dx + \int_A^{5-k} (5 - x - k)\,dx$$



$\ln x = k$ $5 - x = k$

$x = e^k$ $5 - k = x$

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1pt - integrand

1pt - limits

1pt - equation