NO CALCULATOR ALLOWED

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$C'(3.5) = \frac{C(4)-C(3)}{4-3}$$

1pt - approximation

$$= \frac{12.8-11.2}{1}$$

$= 1.6$ ounces/min

1pt - units

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

MVT, cont? diff'able? $C' = \frac{C(b)-C(a)}{b-a}$?

$$C'(t) = \frac{C(4)-C(2)}{4-2}$$

1pt - $\frac{C(4)-C(2)}{4-2}$

$$= \frac{12.8-8.8}{4-2}$$

$$= \frac{4}{2}$$

$$= 2$$

1pt - conclusion using MVT

Since C is cont + diff'able on $(2,4)$, $\exists$ a time t on $(2,4)$ at which $C'(t)=2$.

Continue problem 3 on page 15.

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(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

$\frac{1}{6}\int_0^6 C(t)\,dt = \frac{1}{6}\left[2(5.3) + 2(11.2) + 2(13.8)\right]$ 1pt - mdpt sum

$= \frac{1}{6}\left[2(5.3 + 11.2 + 13.8)\right]$

$= \frac{1}{3}(30.3)$

$= 10.1$ ounces 1pt - approximation

$\frac{1}{6}\int_0^6 C(t)\,dt$ means average # of ounces in the coffee cup from $t = 0$ to $t = 6$ minutes

1pt - meaning w/units

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

$B'(t)$

$B'(t) = -16e^{-.4t} \cdot -.4$ 1pt - $B'(t)$

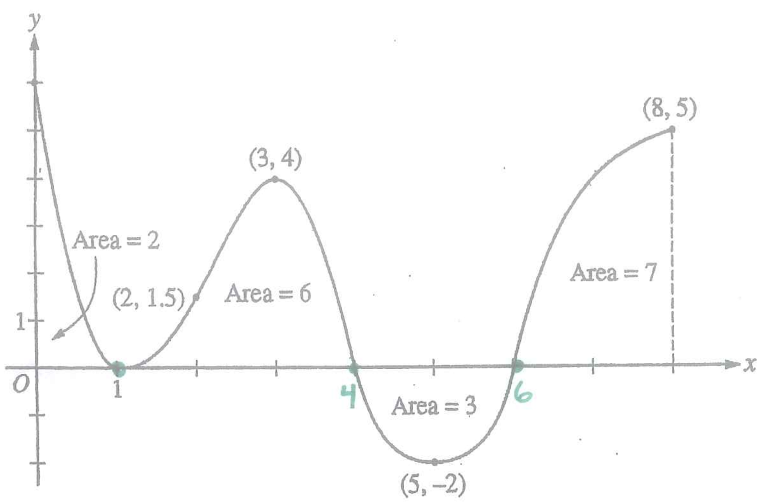
$B'(5) = -16e^{-.4(5)} \cdot -.4$ 1pt - $B'(5)$

$= 6.4e^{-2}$ ounces/min

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

f' + + − +
1 4 6



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$. ← initial value

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

↳ f' ~~pos~~ neg to ~~neg~~ pos.

f has rel. min @ x = 6

b/c f' changes from neg to pos @ x = 6

1pt - answer w/ reason

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

↳ check rel. min + endpts

1pt - consider x = 0 and x = 6

$f(6) = 4 + \int_8^6 f'(x)\,dx = 4 - \int_6^8 f'(x)\,dx$
$= 4 - 7 = -3$

1pt - answer
1pt - reason

$f(0) = 4 + \int_8^0 f'(x)\,dx = 4 - \int_0^8 f'(x)\,dx$
$= 4 - (2 + 6 - 3 + 7)$
$= 4 - (12)$
$= -8$

f(8) = 4

abs min value on [0, 8] is −8

Continue problem 4 on page 17.

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(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

$f'' < 0$ $\quad$ $f' > 0$

f': + + − + on 0, 1, 4, 6, 8 $\qquad$ $f' > 0$ on $(0,4) \cup (6,8)$

f'': − + − + on 0, 1, 3, 5, 8 $\qquad$ $f'' < 0$ on $(0,1) \cup (3,5)$

f concave down and inc on $(0,1) \cup (3,4)$ $\qquad$ 1pt - answer

b/c $f'' < 0$ and $f' > 0$ on $(0,1) \cup (3,4)$ $\qquad$ 1pt - reason

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

$\hookrightarrow g'$

$g'(x) = 3(f(x))^2 \cdot f'(x)$

$g'(3) = 3(f(3))^2 \cdot f'(3)$ $\qquad$ 2pts - $g'(x)$

$= 3\left(-\frac{5}{2}\right)^2 \cdot 4$

$= 3 \cdot \frac{25}{4} \cdot 4$ $\qquad$ 1pt - answer

$= 75$

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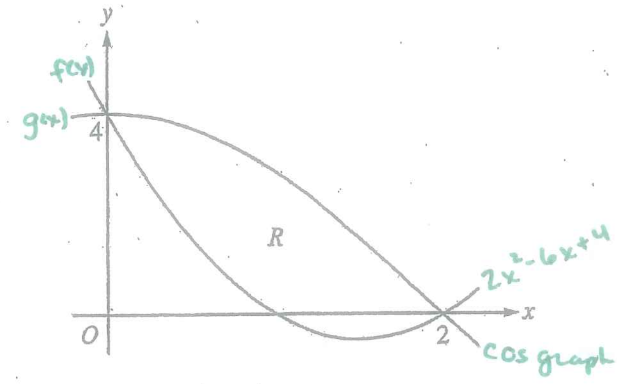

5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

$$\text{Area of } R = \int_0^2 (g(x) - f(x))\,dx$$

1pt- integrand

$$= \int_0^2 \left(4\cos\left(\tfrac{1}{4}\pi x\right) - (2x^2-6x+4)\right) dx$$

$u = \frac{1}{4}\pi x$

$\frac{du}{dx} = \frac{1}{4}\pi$

$\frac{4}{\pi}du = dx$

$u(0) = 0$

$u(2) = \frac{\pi}{2}$

$$= \int_0^{\pi/2} 4 \cdot \cos u \cdot \frac{4}{\pi} du - \int_0^2 (2x^2 - 6x + 4)\,dx$$

$$= \frac{16}{\pi}\int_0^{\pi/2} \cos u\,du - \left(\frac{2}{3}x^3 - 3x^2 + 4x\right)\Big|_0^2$$

2pts antiderivative

$$= \frac{16}{\pi}\sin u\Big|_0^{\pi/2} - \left(\frac{2}{3}(2)^3 - 3(2)^2 + 4(2) - 0\right)$$

$$= \frac{16}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right) - \left(\frac{16}{3} - 12 + 8\right)$$

1pt- answer

$$= \frac{16}{\pi}(1) - \frac{16}{3} + 12 - 8$$

$$= \frac{16}{\pi} - \frac{16}{3} + 4$$

$$= \frac{16}{\pi} - \frac{4}{3}$$

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Continue problem 5 on page 19.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

$$V = \pi \int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2 \right] dx$$

2pts - integrand
1pt - limits + constant

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$\text{Area}_{\text{Square}} = (\text{side})^2$
$= (g(x) - f(x))^2$

$$V = \int_0^2 (g(x) - f(x))^2 \, dx$$

1pt - integrand
1pt - limits + constant

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NO CALCULATOR ALLOWED

6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

$y - y_1 = m(x - x_1)$

$\left.\frac{dy}{dx}\right|_{(1,0)} = e^0(3(1)^2 - 6(1))$

$= 1(3-6)$

$= -3$

1pt - $\left.\frac{dy}{dx}\right|_{(1,0)}$

$y - 0 = -3(x-1)$

1pt - tangent line

$y = -3(1.2 - 1)$

$y = -3(.2)$

$y = -.6$

$f(1.2) = -.6$

1pt - approximation

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Do not write beyond this border.

Continue problem 6 on page 21.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

↳ y's w/ dy's, x's w/ dx's

$$\frac{dy}{dx} = e^y(3x^2 - 6x)$$

$$dy = e^y(3x^2 - 6x)\,dx$$

$$\int \frac{1}{e^y}\,dy = \int (3x^2 - 6x)\,dx$$

1 pt – separate variables

$$\int e^{-y}\,dy = x^3 - 3x^2 + c$$

$u = -y$

$\frac{du}{dy} = -1$

$-du = dy$

$$\int e^u \cdot -1\,du = x^3 - 3x^2 + c$$

$$-\int e^u\,du = x^2 - 3x + C$$

$$-e^u = x^2 - 3x + C$$

$$-e^{-y} = x^2 - 3x + C$$

2 pts – antiderivative

1 pt – "+C"

$$-e^{-(0)} = 1^2 - 3(1) + C$$

1 pt – initial condition

$$-e^0 = -2 + C$$

$$-1 = -2 + C$$

$$1 = C$$

$$-e^{-y} = x^2 - 3x + 1$$

$$\ln\ e^{-y} = -x^2 + 3x - 1\ \ln$$

$$-y = \ln|-x^2 + 3x - 1|$$

$$y = -\ln|-x^2 + 3x - 1|$$

1 pt – solve for y

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